1949

Eddy current damping in flat and cylindrical conducting surfaces.

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Princeton University

ESSAY

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EDDY CURRENT DAMPING IN FLAT AND CYLINDRICAL CONDUCTING SURFACES

by

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An Essay

Submitted to the Advisory Board of

The School of Engineering,

The Johns Hopkins University

In Conformity with the Requirements for

The Degree of Master of Engineering

Baltimore, Maryland

May 1949

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TABLE OF CONTENTS

TABLE OF CONTENTS

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EDDY CURRENT DAMPING
IN FLAT AND CYLINDRICAL CONDUCTING SURFACES

SECTION I
FOREWORD

The use of eddy currents to produce damping in dynamical systems is becoming more prevalent than heretofore, particularly in the design of accelerometers and similar instruments. The value of eddy current damping lies in the fact that the force produced is more nearly proportional to velocity undermuch wider conditions of damping than with other systems. The chief disadvantage of the use of eddy current damping is the size and weight of the magnet which is required. If an electromagnet is used there must be a source of well regulated electric current available; if a permanent magnet is used it must maintain a substantially constant value of gap flux under all operating conditions and for a long period of time. Instruments for use in airborne craft in particular have strict requirements as to size and weight.

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These disadvantages are largely being overcome by the use of high-retentivity magnetic materials such as Alnico and should present less of a problem with the passage of time.

The physical arrangement for producing eddy current damping can take a great variety of forms. It is the purpose of this paper to analyze two basic types which have perhaps the greatest possibility of use. The analysis will then be substantiated with experimental data. The two types to be considered are:

A -- A flat conducting plate moving linearly between the poles of a magnet in a direction perpendicular to the lines of magnetic flux.

B -- A conducting cylinder moving linearly in a direction perpendicular to the lines of flux of a radial field; the axis of the radial field being coincident with the axis of the cylinder and the field being uniform and finite between two parallel planes which are perpendicular to the axis of the cylinder.

Type A can be called a "flat plate" system and will be here subdivided as to whether the poles of the magnet have a rectangular or circular cross section. Type B can be called a "cylindrical"

system. It is sometimes called a "cup" system when one end of the conducting cylinder is covered over for mounting purposes. It will be brought out that the covering of one end of the cylinder is bad practice and hence the term "cup" will not be used for the general classification. The two types of systems are useful because they are both simple as to design and construction. Other more complicated systems are possible. The methods used here should apply in principle, if not directly, to nearly all systems having engineering applications, and particularly in the field of instrumentation.

The physical laws upon which eddy current damping is based are well known and will not be discussed here. The principle involved is the production of electric currents in a conducting medium when that medium is in motion in a changing magnetic field, and the resulting force produced on the conducting medium by the interaction of these currents with the causative field. For clarity a listing of the formulae, together with definitions of the symbols to be used follows:

$\underline{E} = \text{grad } e$

$e = [\underline{v} \times \underline{B}] \cdot \underline{dl}$

$\underline{I} = \sigma \underline{E}$

$d\underline{I}_p = \dfrac{[\underline{\text{curl } I} \times \underline{r}]}{2\pi r^2}$

$d\underline{f}_p = [\underline{I}_p \times \underline{B}]\, dV$

$d\underline{f}' = d\underline{f} \cos \angle_f^v$

$\underline{F} = \int d\underline{f}'$

e = electric potential difference

E = electric potential gradient

B = magnetic flux density

I = current density

σ = conductivity of conducting medium

D,R,r,r',a,b=linear distances

f=force at point P

f'=force at point P in a direction parallel to the direction of motion of the conducting medium

F =total force on the conducting medium

v =linear velocity of the conducting medium relative to the magnetic field

d =thickness of conducting medium

Φ =total magnetic flux

A =area

V =volume

dl=element of length

i,j,k=unit vectors, right handed orthogonal system

x,y,z=cartesian coordinate axes

All quantities treated as vectors are underlined.

All quantities are in c.g.s. units.

The following assumptions have been made for the analytical development:

1. The flux density is uniform and constant between the magnet poles and zero everywhere else (no fringing effect).

2. The thickness, d, of the conducting medium is small enough in relation to the other dimensions

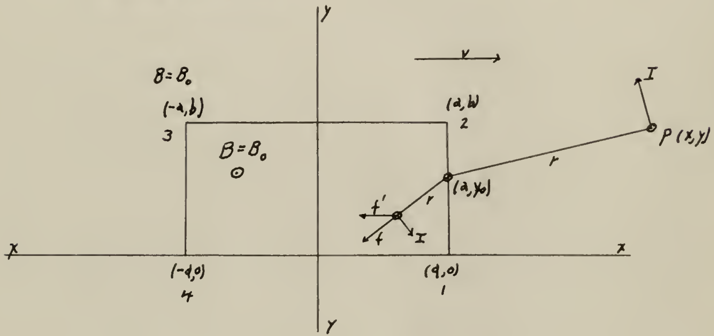
so that for analytical purposes the flat plate can be considered as a plane and the cylinder as a bent plane.

3. The conducting medium contains no ferromagnetic materials. This assumption must be rigidly adhered to in practice since even very small amounts of magnetic material in the conducting medium will prevent the damping force from being linear with respect to velocity, and an entirely new analysis will be required.

SECTION II

MATHEMATICAL ANALYSIS OF THE VARIOUS SYSTEMS

A-1 Flat plate moving between the poles of a magnet having pole faces rectangular in cross section.



Consider a flat conducting plate of infinite x y dimensions and thickness d in the z direction, moving with a velocity v toward the right between the poles of a magnet which have total linear dimensions of 2a and b.

From the sketch above, which shows the projection of the pole face on the plate, there will be a change of flux along the lines 1-2 and 3-4, but none along 2-3 or 4-1.

For the right side of the pole face, line 1-2:

$$e = [\underline{v} \times \underline{B}] \cdot \underline{dl}$$

$$= [v\underline{i} \times B_0 \underline{k}]\, dy_0 \underline{j}$$

$$\underline{E} = -v B_0 \underline{j}$$

$$\underline{\text{curl}\,E} = [\underline{\nabla} \times \underline{E}] = -v B_0\, dy_0 \underline{k}$$

$$\underline{\text{Curl } I} = \sigma v B_0\, dy_0 \underline{k}$$

$$\underline{r} = (x-a)\underline{i} + (y-y_0)\underline{j}$$

$$[\underline{\text{curl}\,I} \times \underline{r}] = \sigma v B_0 [-(y-y_0)\underline{i} + (x-a)\underline{j}]\, dy_0$$

$$r^2 = (x-a)^2 + (y-y_0)^2$$

$$(d\underline{I}_{xy})_R = \frac{-B_0 \sigma v}{2\pi} \left[\frac{(y-y_0)\underline{i} - (x-a)\underline{j}}{(x-a)^2 + (y-y_0)^2} \right] dy_0$$

$$(\underline{I}_{xy})_R = \frac{B_0 \sigma v}{2\pi} \left[-\underline{i} \int_0^b \frac{(y-y_0)\, dy_0}{(x-a)^2 + (y-y_0)^2} + \right.$$

$$\left. \underline{j} \int_0^b \frac{(x-a)\, dy_0}{(x-a)^2 + (y-y_0)^2} \right]$$

For the left side of the pole face, line 3-4:

$$\underline{\text{curl } I} = -\sigma v B_0 dy_0 \underline{k}$$

$$\underline{r} = (x+a)\underline{i} + (y-y_0)\underline{j}$$

$$[\underline{\text{curl } I} \times \underline{r}] = -\sigma v B_0 [-(y-y_0)\underline{i} + (x+a)\underline{j}]\, dy_0$$

$$r^2 = (x+a)^2 + (y-y_0)^2$$

$$(d\underline{I}_{xy})_L = \frac{B_0 \sigma v}{2\pi} \left[\frac{(y-y_0)\underline{i} - (x+a)\underline{j}}{(x+a)^2 + (y-y_0)^2} \right] dy_0$$

$$(\underline{I}_{xy})_L = \frac{B_o \sigma v}{2\pi}\left[\underline{i}\int_0^b \frac{(y-y_o)dy_o}{(x+a)^2+(y-y_o)^2} - \underline{j}\int_0^b \frac{(x+a)dy_o}{(x+a)^2+(y-y_o)^2}\right]$$

$$\underline{I}_{xy} = (\underline{I}_{xy})_R + (\underline{I}_{xy})_L$$

Performing the indicated integrations and combining their results:

$$\underline{I}_{xy} = \frac{B_o \sigma v}{2\pi}\left\{\left[-\frac{1}{2}\ln\frac{(x-a)^2+(y-b)^2}{(x-a)^2+y^2} - \frac{1}{2}\ln\frac{(x+a)^2+(y-b)^2}{(x+a)^2+y^2}\right]\underline{i} + \left[\tan^{-1}\frac{y}{x-a} - \tan^{-1}\frac{y-b}{x-a} - \tan^{-1}\frac{y}{x+a} + \tan^{-1}\frac{y-b}{x+a}\right]\underline{j}\right\}$$

This expression gives the current density at any point P(x,y) in the infinite plane in terms of its normal components.

To find the force on the moving plate produced by this current density distribution:

$d\underline{f} = \left[\underline{I}x\underline{B}\right]$ dx dy d, which is the force on an elemental volume dx dy d.

Let $\underline{I}_{xy} = K\left[L\underline{i}+M\underline{j}\right]$ as a general expression

Then:

$$d\underline{f} = \left[K(L\underline{i}+M\underline{j})xB_o\underline{k}\right] dxdyd$$

$$d\underline{f} = K(MB_o\underline{i}-LB_o\underline{j})\ dxdyd$$

and $\quad d\underline{f}' = KMB_o\underline{i}dxdyd$

$$\underline{F} = \int d\underline{f}' = KB_o d\underline{i} \int_0^b dy \int_{-a}^{a} Mdx$$

Force is produced only within the area of the plate covered by the pole faces since $B = B_o$ within this area and $B = 0$ outside of this area:

Performing the integrations:

$$\underline{F} = B_o^2 \sigma vd2ab \left\{ \frac{1}{\pi} \left[2\tan^{-1} \frac{b}{2a} - \frac{1}{2} \frac{2a}{b} \ln\left(1 + \frac{b^2}{4a^2} \right) + \frac{1}{2} \frac{b}{2a} \ln\left(1 + \frac{4a^2}{b^2}\right)\right] (-\underline{i}) \right\}$$

The term $(-\underline{i})$ indicates that the force is in the direction parallel but opposite to the direction of movement of the plate. The $\underline{j}$ term in the expression for df has a value of zero showing that there are no forces tending to move the plate in a perpendicular (y) direction. The $\underline{k}$ term which might result in a force parallel to the lines of flux is completely absent. The term in bars $\{\ \}$ in the expression for $\underline{F}$ is a dimensionless term whose value depends on the ratio of lengths of the sides of the pole face. Calling this term C, and noting that 2ab equals the area of the pole face A, then:

$$F = B_o^2 \sigma vdAC$$

or

$F = \Phi^2 \sigma v d A^{-1} C$ where $B_0 A = \Phi$, the total flux.

The variation of the factor C as the ratio of the lengths of the sides of the pole face varies is shown in A-1. Thus, other factors such as convenient physical arrangement not considered, it is seen that the pole faces should be longer in the direction perpendicular to the direction of motion of the plate (y) than in the parallel direction (x).

The development thus far has been for a plate which has infinite x and y dimensions. Of course in actual application the dimensions of the plate must be finite. In order to account for this fact let the expression for the force on the plate, F, be multiplied by a factor C'.

$$F = \Phi^2 \sigma v d A^{-1} C C'$$

The object of the following discussion will be to determine the value of C' for various configurations of the finite plate.

It is desired first to plot the curves of constant current density for an infinite plate with a given pole face configuration. This plot could be made from the equation:

$$\underline{I}_{xy} = K\left[L\underline{i} + M\underline{j}\right]$$

The following development, however, gives an easier expression with which to work:

$$|\text{curl } I| = \pm |v\sigma B_o dl|$$

$$|(dI)_R| = \frac{|B_o \sigma v dl|}{2\pi |r|}, \quad |r| = \sqrt{(x-a)^2 + (y-y_o)^2}$$

$$|(dI)_L| = \frac{|B_o \sigma v dl|}{2\pi |r|}, |r| = \sqrt{(x+a)^2 + (y-y_o)^2}$$

$$|I_R| = \frac{|B_o \sigma v|}{2\pi} \int_0^b \frac{dy_o}{\sqrt{(x-a)^2 + (y-y_o)^2}}$$

$$|I_L| = \frac{|B_o \sigma v|}{2\pi} \int_0^b \frac{dy_o}{\sqrt{(x-a)^2 + (y-y_o)^2}}$$

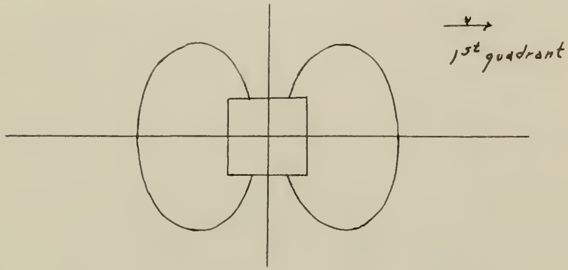
$$|I| = |I_R| + |I_L|$$

Performing the integrations and combining:

$$|I| = \frac{|B_o \sigma v|}{2\pi} \ln \left(\left[\frac{(y-b) + \sqrt{(x+a)^2 + (y-b)^2}}{(y-b) + \sqrt{(x-a)^2 + (y-b)^2}} \right] \left[\frac{y + \sqrt{(x-a)^2 + y^2}}{y + \sqrt{(x+a)^2 + y^2}} \right] \right)$$

From the expression the constant current density curves have been plotted for a square pole face, Figure A-2, and a pole face which is three times as long in a direction perpendicular to the direction of motion of the plate (y) as in the parallel direction(x), Figure A-6. The factor $\frac{|B_o \sigma v|}{2\pi}$ has been left off so that the values of the constant current density curves indicate relative

magnitude only. Figures A-2 and A-6 show the first quadrant only of the infinite xy plane, the other quadrants having reflection symmetry with respect to the axes of symmetry of the pole face.

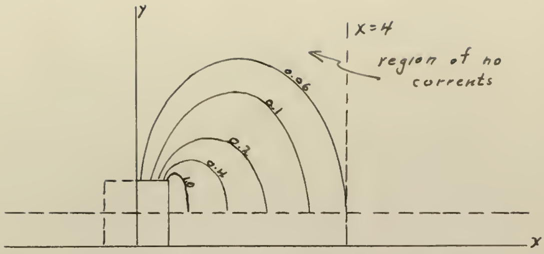


For Figure A-2 the unit of distance is 2a (also equal to b in this case). For Figure A-6 the unit of distance is also 2a. The ordinate for Figure A-6 starts at 1.5 which is not the analytic x axis, but rather the axis of symmetry of the pole face.

If now the plate were made finite the shape and relative magnitude of these curves would be altered in some manner such that the current density would be zero at all boundaries of the plate. A true solution

of this condition involves a field problem beyond the scope of this paper. As an alternative let it be assumed that the curves of constant current density maintain their shape and relative magnitude regardless of the boundaries of the plate so long as a curve does not cross a boundary, in which case that particular curve then takes on a value of zero.

For purposes of illustration consider the curves for a square pole face, Figure A-2. Let the plate be terminated along the line $x=4$ (no termination in the y direction). According to the assumption made above all current density curves having a value $\geq$ 0.06 still exist in their former shape, but all curves having a value $<$ 0.06 reach the edge of the plate at some point along the line $x=4$ and hence their value now becomes zero. As a physical justification for the assumption if it is considered that the constant current density curves each represent a conductor carrying the specified relative amount of current, there will then be no currents of relative magnitude $<$ 0.06 since these conductors are now "open circuited".



This same reasoning can be immediately applied to plate terminations along any other value of x, y, or any other plate configuration.

Under the considerations discussed above, it is seen that the total current in the plate passing between the pole faces has been reduced from infinite plate conditions. Since the force produced on the plate is directly proportional to this total current ($f = \left[I \times B_o \right] dV$), then the force will be reduced accordingly. It is seen then that the factor C' has a value of unity for an infinite plate and a value less than unity for a finite plate.

In order to obtain the amount of total current reduction a curve is drawn giving the relative current density in the plate as a function of x along a line $y = b/2$, the symmetry axis of the pole face. This curve for the square pole face is shown on Figure A-3 for that portion of the plate where $x > 0.5$, the edge of the pole face. C' will now be the ratio of the area under this curve from 0.5 to a given value of x to the total area under the curve.

In this case the following is used:

$$I \propto \ln\left[\frac{-0.5+\sqrt{(x+0.5)^2+0.25}}{-0.5+\sqrt{(x-0.5)^2+0.25}}\right]\left[\frac{0.5+\sqrt{(x-0.5)^2+0.25}}{0.5+\sqrt{(x+0.5)^2+0.25}}\right]$$

so that:

$T = \int_{0.5}^{x} I\,dx$ where T is "total current" flowing in the plate to the right of $x=0.5$, and consequently under the right half of the pole face.

T is evaluated as

$x \rightarrow x$

$x \rightarrow \infty$

and

$$C' = \frac{T_x}{T_\infty}$$

The curve of C' versus x, where x indicates a plate termination along a line x=constant is given in Figure A-4.

Now according to the procedure outlined, if the plate be terminated along any line of arbitrary shape then the line of minimum constant current density which is left whollyintact will be equivalent to a termination along a line x=constant at that particular value of constant current density. Thus, for example, considering

Figure A-2, a plate termination along y=3.1 is equivalent to a termination along x= 4, and similarly for any other termination which leaves the 0.06 line intact but cuts off all lines of lesser value in at least one spot (i.e., opens the circuit). For clarity it is again stated that Figure A-2 shows the entire height, b, of the pole face but only half of the width, a, in accordance with the system of coordinates used.

So far the discussion has dealt with termination of the plate in the first quadrant only. Since all constant current density lines of Figure A-2 are closed on themselves through the 4th quadrant any plate termination in either the first or fourth quadrants will affect these lines. The same is true for the 2nd and 3rd quadrants, but it is to be noted that the 1st and 4th quadrant currents are independent of those in the 2nd and 3rd quadrants. If the plate termination is symmetrical with respect to the pole face, as it would normally be; that is, the plate is terminated at equal distances in the ± x direction and at equal distances in the ± y direction with respect to the geometrical center of the pole face, then the values of C' from Figure A-4 can be used directly in the equation for force. If the

termination is not symmetrical then the forces produced on each side of the center line of pole face ($\pm x$) must be calculated by applying C' to each side individually. Since the force produced on either half from the center line is just half of the total force for an infinite plate, and since the two halves are assumed independent of each other, the required calculation can be made as follows:

$$F_{T_F} = F_{R_I} C'_R + F_{L_I} C'_L$$

$$F_{T_F} = \frac{1}{2} F_{T_I} C'_R + \frac{1}{2} F_{T_I} C'_L$$

$$F_{T_F} = F_{T_I} \left(\frac{C'_R + C'_L}{2} \right)$$

where: F_{T_F} = total force on the finite plate

F_{T_I} = total force on the infinite plate

C'_R = C' for the right half of the plate

C'_L = C' for the left half of the plate

F_{R_I} = force which would be produced on the right half of the infinite plate

F_{L_I} = force which would be produced on the left half of the infinite plate

According to the theory developed if a plate were terminated at $x = 4$ and $y = 3.1$ then the area of the plate

outside of the constant current density line 0.06 would have no currents in it and hence would not be contributing to damping. In other words the plate would be carrying along excess mass. In order to achieve the maximum effectiveness (i.e. greatest force for the least amount of plate) the plate should be terminated entirely along the line 0.06 for the example being used. A termination such as described would result in a plate of figure 8 shape which is rather impractical. Also, the amplitude of total motion of the plate when used in vibrating systems has been assumed small in comparison with the dimensions of the plate, and while this is a not impractical assumption, it would rule out cutting the plate to the exact shape. The next best type of termination would then be one giving an oval shape; the ends of the plate following the desired curve and the sides being level along the horizontal (y=constant) tangents to the curve. For a plate of this shape, it is seen that the length in the direction of motion (x) should be greater than the length in the direction perpendicular to the motion (y). Figure A-5 shows this relationship.

Figures A-6, A-7, A-8, A-9 show the characteristics

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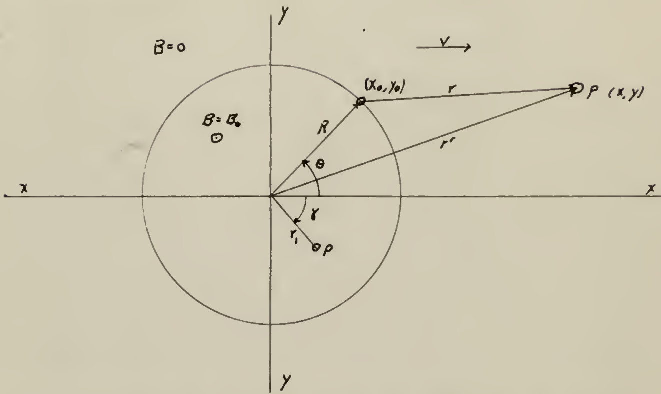
for a rectangular pole face of dimensions 1 unit in the direction of motion (x) and 3 units perpendicular to the motion (y). They correspond to Figures A-2, A-3, A-4, A-5 respectively for the square pole face. All the discussion relative to the square pole face is directly applicable.

It will be part of the experimental work to check the validity of the assumptions made above.

.-2 Flat plate moving between the poles of a magnet having pole faces circular in cross section.



The arrangement in this case is similar to that for rectangular pole faces as indicated in the figure above with its accompanying notation.

$$\underline{r} = \underline{r}' - \underline{R}$$

$$\underline{r}' = x\underline{i} + y\underline{j}$$

$$\underline{R} = x_0\underline{i} + y_0\underline{j}$$

$$\underline{r} = (x-x_0)\underline{i} + (y-y_0)\underline{j}$$

$$r^2 = (x-x_0)^2 + (y-y_0)^2$$

$$dl = Rd\theta\ (-\sin\theta\underline{i} + \cos\theta\underline{j})$$

For the right side of the pole face, $-\pi/2 < \theta < \pi/2$:

$$\underline{\text{curl}}\ \underline{I} = \sigma v B_o \frac{x_o}{R} R d\theta\, \underline{k}$$

$$[\underline{\text{curl}}\ \underline{I} x \underline{r}] = \sigma v B_o [-(y-y_o)\underline{i} + (x-x_o)\underline{j}]\, x_o d\theta$$

$$(d\underline{I}_{xy})_R = \frac{-B_o \sigma v}{2\pi}\left[\frac{(y-y_o)\underline{i}-(x-x_o)\,\underline{j}}{(x-x_o)^2+(y-y_o)^2}\right] x_o d\theta$$

$$(\underline{I}_{xy})_R = \int_{-\pi/2}^{\pi/2} (d\underline{I}_{xy})_R$$

For the left side of the pole face, $-\pi/2 > \theta > \pi/2$:

$$\underline{\text{curl}}\ \underline{I} = \sigma v B_o \frac{x_o}{R} R d\theta\, \underline{k}$$

$$[\underline{\text{curl}}\ \underline{I} x \underline{r}] = \sigma v B_o [-(y-y_o)\underline{i} + (x-x_o)\underline{j}]\, x_o d\theta$$

$$(d\underline{I}_{xy})_L = \frac{-B_o \sigma v}{2\pi}\left[\frac{(y-y_o)\underline{i}-(x-x_o)\underline{j}}{(x-x_o)^2+(y-y_o)^2}\right] x_o d\theta$$

$$(\underline{I}_{xy})_L = \int_{\pi/2}^{-\pi/2} (d\underline{I}_{xy})_L$$

$$\underline{I}xy = (\underline{I}_{xy})_R + (\underline{I}_{xy})_L$$

Expressing: $x_o = R\cos\theta$

$y_o = R\sin\theta$

Then I can be written as:

$$\underline{I}_{xy} = \frac{-B_o \sigma v R}{2\pi}\left[\underline{i}\int_0^{2\pi} \frac{(y-R\sin\theta)\cos\theta\ d\theta}{x^2+y^2+R^2-2Rx\cos\theta-2Ry\sin\theta}\right.$$

$$\left. -\underline{j}\int_0^{2\pi} \frac{(x-R\cos\theta)\cos\theta\ d\theta}{x^2+y^2+R^2-2xR\cos\theta-2Ry\sin\theta}\right]$$

In general terms;

$$\underline{I}_{xy} = \frac{-B_o \sigma vR}{2\pi}\left[L\underline{i} - M\underline{j}\right]$$

$$d\underline{f} = \left[\underline{I}_{xy} \times \underline{B}\right] dV$$

Now within the area of the pole faces the element of volume is $(d\ r_1\ dr_1\ d\gamma)$ and $x = r_1\cos\gamma$, $y = r_1\sin\gamma$, so that:

$$d\underline{f} = \frac{B_o \sigma vR}{2\pi}\left\{\left[L\underline{i} - M\underline{j}\right] \times B_o\underline{k}\right\} d\ r_1 dr_1 d\gamma$$

$$d\underline{f} = \frac{B_o^2 \sigma vR}{2\pi}\left[-M\underline{i} - L\underline{j}\right] d\ r_1 dr_1 d\gamma$$

$$d\underline{f}' = \frac{B_o^2 \sigma vdRr_1}{2\pi} M\underline{i}\, dr_1 d\gamma$$

$$\underline{F} = \int d\underline{f}' = \frac{B_o^2 \sigma vRd}{2\pi}\,\underline{i}\int_0^R r_1 dr_1 \int_0^{2\pi} Md\gamma$$

Performing the indicated integrations the triple integral is found to have a value of $-\pi^2 R$

$$\underline{F} = \frac{B_o^2 \sigma vRd\,\underline{i}}{2\pi}\left(-\pi^2 R\right)$$

$$\underline{F} = \frac{1}{2} B_o^2 \sigma vd\,\pi R^2\,(-\underline{i})$$

$$\underline{F} = \frac{1}{2} B_o^2 \sigma vdA\,(-\underline{i})$$

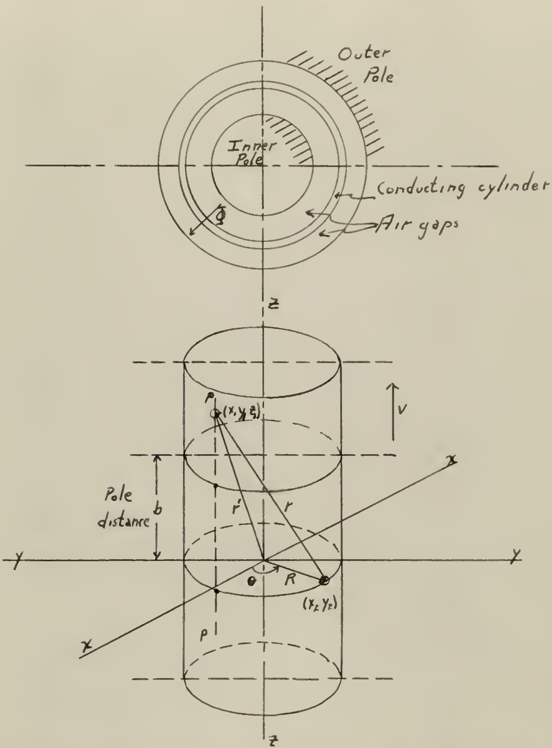
$F = \frac{1}{2}\Phi^2 \sigma\, vdA^{-1}$ in the direction opposite to the motion of the plate.

The expression developed for F is similar to that in the case of the rectangular pole face except that the

factor C now has a constant value of 0.5. Dimensionally a relationship such as this is expected. C is a dimensionless factor dependent on the two parameters which express the configuration of the rectangular pole face. Since a circle is expressed by only one parameter, R, there could be no such expression for the circular pole face. From a quantitative point of view the factor of 0.5 corresponds to the value of 0.5 for C obtained when a square pole face is used.

A development of the curves of constant current density analogous to those made for the rectangular pole face will not be made here. These curves will be made from the experimental data in Section III. The discussion relative to the curves for the rectangular pole face will then apply for this case.

B Cylindrical conductor moving in a radial field.



Consider a conducting cylinder of infinite length in the z direction, radius R, and wall thickness d, moving through a radial field of flux. The flux field has a center line coinciding with the axis of the cylinder and is uniform over a distance b along the axis of the cylinder. Above and below this pole distance, b, the

field is zero. Using the notation according to the figure above, and following the same procedure as discussed with the flat plate, the analysis is as follows:

For the lower pole surface:

$$\underline{\text{curl } I} = -\sigma v B_o(x_2\underline{i} + y_2\underline{j}) \Big/ \sqrt{x_2^2 + y_2^2}\; Rd\theta$$

$$\underline{R} = x_2\underline{i} + y_2\underline{j}$$

$$\underline{r}' = x_1\underline{i} + y_1\underline{j} + z_1\underline{k}$$

$$\underline{r} = \underline{r}' - \underline{R} = (x_1 - x_2)\underline{i} + (y_1 - y_2)\underline{j} + z_1\underline{k}$$

$$[\underline{\text{curl } I} \times \underline{r}] = -\sigma v B_o\left[y_2 z_1 \underline{i} - x_2 z_1 \underline{j} + (x_2 y_1 - y_2 x_1)\underline{k}\right] \Big/ \sqrt{x_2^2 + y_2^2}\quad Rd\theta$$

$$r^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2 + z_1^2$$

$$(d\underline{I}_{xyz})_L = \frac{-vB_o\sigma}{2\pi\sqrt{x_2^2 + y_2^2}}\left[\frac{y_2 z_1 \underline{i} - x_2 z_1 \underline{j} + (x_2 y_1 - y_2 x_1)\underline{k}}{(x_1 - x_2)^2 + (y_1 - y_2)^2 + z_1^2}\right] Rd\theta$$

From the symmetry of the arrangement the current density distribution along any line parallel to the axis of the cylinder will be the same as that along any other parallel line. Let an arbitrary point along one of these parallel lines be given by (R, o, z_1).

Then:

$$x_1^2 + y_1^2 = R^2$$

$$x_2^2 + y_2^2 = R^2$$

$$x_1 = R$$

$$y_1 = o$$

$$(d\underline{I}_{R,0,z_1})_L = \frac{-B_o \sigma v R^2}{2\pi R} \left[\frac{z_1(\sin\theta\, \underline{i} - \cos\theta\, \underline{j}) - R\sin\theta\, \underline{k}}{2R^2 + z_1^2 - 2R^2\cos\theta} \right] d\theta$$

$$(\underline{I}_{R0z_1})_L = \frac{-B_o \sigma v R}{2\pi} \left[z_1 \underline{i} \int_0^{2\pi} \frac{\sin\theta\, d\theta}{D} - z_1 \underline{j} \int_0^{2\pi} \frac{\cos\theta\, d\theta}{D} - R\underline{k} \int_0^{2\pi} \frac{\sin\theta\, d\theta}{D} \right]$$

where D expresses the denominator of the bracketed expression of $d\underline{I}$. The $\underline{i}$ and $\underline{k}$ integrals have a value of zero so that:

$$(\underline{I}_{R0z_1})_L = \frac{B_o \sigma v R \underline{j} z_1}{2\pi} \int_0^{2\pi} \frac{\cos\theta\, d\theta}{2R^2 + z_1^2 - 2R^2\cos\theta}$$

and

$$(\underline{I}_{R0z_1})_L = \frac{B_o \sigma v}{2R} \left[z_1 - \frac{2R^2 + z_1^2}{\pm\sqrt{4R^2 + z_1^2}} \right] (-\underline{j})$$

The negative sign in front of the radical is to be used whenever z_1 is negative in the indicated coordinate system.

For the upper surface the analysis is the same except that the whole expression will be the negative of the one for the lower surface and all z_1s are replaced by $(z_1 - b)$.

$$(\underline{I}_{R0z_1})_U = \frac{B_o \sigma v}{2R} \left[z_1 - b - \frac{2R^2 + (z_1 - b)^2}{\pm\sqrt{4R^2 \; (z_1 \; b)^2}} \right] (\underline{j})$$

Again, the negative sign in front of the radical is to be used whenever $(z_1 - b)$ is negative with respect to the upper pole surface.

$$\underline{I}_{R0z_1} = (\underline{I}_{R0z_1})_L + (\underline{I}_{R0z_1})_U$$

$$\underline{I}_{R0z_1} = \frac{B_o \sigma v}{2R}\left[-b + \frac{2R^2 + z_1^2}{\pm\sqrt{4R^2 + z_1^2}} - \frac{2R^2 + (z_1 - b)^2}{\pm\sqrt{4R^2 + (z_1 - b)^2}}\right]\underline{j}$$

The complete expression for the current density distribution at any point on the cylinder determined by z_1 and θ is:

$$I_{z_1\theta} = \frac{B_o \sigma v}{2R}\left[-b + \frac{2R^2 + z_1^2}{\pm\sqrt{4R^2 + z_1^2}} - \frac{2R^2 + (z_1 - b)^2}{\pm\sqrt{4R^2 + (z_1 - b)^2}}\right](-\sin\theta\,\underline{i} + \cos\theta\,\underline{j})$$

This expression shows that all lines of constant current density are circular with respect to the axis of the conducting cylinder. Thus all lines are closed on themselves and the cylinder represents an infinite plane in the light of the flat plate analysis.

Proceeding as before to calculate the force produced on the cylinder:

$$d\underline{F} = \left[\,\underline{I} \times \underline{B}\,\right] dV$$

$$\underline{B} = B_o(\cos\theta\,\underline{i} + \sin\theta\,\underline{j})$$

$$dV = dR\,d\theta\,dz_1$$

Performing the indicated operations there results:

$$d\underline{F} = \frac{1}{2} B_o^2 \sigma v d(-\underline{k}) \left[- b + \frac{2R^2 + z_1^2}{\pm\sqrt{4R^2 + z_1^2}} - \frac{2R^2 + (z_1 - b)^2}{\pm\sqrt{4R^2 + (z_1 - b)^2}} \right] d\theta dz_1$$

$$\underline{F} = \frac{1}{2} B_o^2 \sigma v d(-\underline{k}) \left\{ \int_0^b \left[- b + \frac{2R^2 + z_1^2}{\sqrt{4R^2 + z_1^2}} + \frac{2R^2 + (z_1 - b)^2}{\sqrt{4R^2 + (z_1 - b)^2}} \right] dz_1 \int_0^{2\pi} d\theta \right\}$$

Note that the radicals now take their plus sign since in the coordinate system used z_1 does not go negative between 0 and b and $(z_1 - b)$ does not go positive with respect to the upper pole surface in the interval.

Performing the integration:

$$\underline{F} = B_o^2 \sigma \, v d \pi \left[b\sqrt{4R^2 + b^2} - b^2 \right] (-\underline{k})$$

The term in brackets will always be positive so that the force is always in the direction opposite to the motion of the conducting cylinder.

Rearranging terms:

$$F = B_o^2 \sigma \, v d 2\pi \, Rb \left[\sqrt{1 + b^2/4R^2} - \frac{b}{2R} \right]$$

$$F = B_o^2 \sigma \, v d A C$$

where $C = \sqrt{1 + \left(\frac{b}{D}\right)^2} - \frac{b}{D}$

A = mean area of the cylinder enclosed

[illegible]

$$[illegible]$$

$$[illegible]$$

[illegible]

[illegible]

$$[illegible]$$

[illegible]

[illegible]

$$[illegible]$$

[illegible]

within the pole distance.

D=mean diameter of the cylinder.

$$F = \Phi^2 \sigma v d C A^{-1}$$

The curve of C versus $\frac{b}{D}$, Figure A-12, shows that C approaches a maximum value of 1. For practical application values of C on the order of 0.7 should not be difficult to obtain. Since all the circulating currents are closed on themselves the value of C', if such a parameter should be used, is always unity. Thus for this arrangement the force is not dependent on the length of the cylinder outside of the region of flux, except that for use in vibrating systems it should be long enough to cover the pole faces plus twice the maximum amplitude of movement.

A comparison between the cylindrical and flat plate systems is difficult to make. If, however, the quantities Φ, A, d, and σ are considered to be the same for each, then some comparison can be made by examining only the factors C and C'. For the cylindrical system a practical value of C might be 0.7. (C'= 1) For the rectangular pole face values of C=0.7 and C'= 0.7 seem reasonable, then giving 0.5 for their product. For the circular pole face C' = 0.7. (C = 0.5)

giving the combined factor a value of 0.35. From this elementary comparison, the cylindrical system appears best. In addition, since the cylinder need have only enough metal to cover the poles, its mass would be expected to be less than that of the flat plate types. When used in vibrating dynamical systems this lesser mass requires less force for the same amount of damping so that with the cylindrical type not only is more force produced but less force is required. Perhaps the greatest utilization of this increased effectiveness would be in permitting a smaller magnet to be used; that is, smaller in size, weight and current requirements if it is of that type.

Figure A-10 shows the variation in current density from one "generating surface" as the radius of the cylinder is changed. The area under any one curve is a measure of the total current flowing.

Figure A-11 combines two "generating surfaces" to show the resultant current density distribution.

SECTION III

EXPERIMENTAL

A. General discussion of the experimental procedure.

In Section II the following formulas were developed:

1. For a flat plate moving between the poles of a magnet, the pole faces being rectangular in shape:

$$F = \Phi^2 \sigma v d A^{-1} C C'$$

2. For a flat plate moving between the poles of a magnet, the pole faces being circular in shape:

$$F = \tfrac{1}{2} \Phi^2 \sigma v d A^{-1} C'$$

3. For a cylindrical conductor moving in a radial magnetic field:

$$F = \Phi^2 \sigma v d A^{-1} C$$

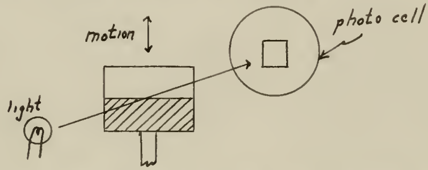
The object of the various experiments performed was to verify that the equations for force, as given above, do actually contain the factors therein; that the factors C and C' vary according to the analysis developed in Section II; and that the force equation for a cylindrical conductor does actually lack a factor C'.

The general procedure followed in the experiments was to hold constant as many of the factors as possible and vary the one for which the experiment was being run. For those factors which were not able to be kept constant a corrective adjustment was made in analyzing that particular test. These corrective adjustments will be described for each test.

The device used for the experiments was a mass-spring system which was subject to eddy current damping. For all flat plate experiments the mass consisted of a copper plate suspended by two thin flat springs. The suspension was such that the motion of the plate was very nearly linear and no force on the plate due to gravity (i.e. as the restoring force on a pendulum) has been used. The magnetic field through which the plate was moved was produced by a ring of round iron stock, containing an air gap, and wound with a number of turns of wire to form an electro-magnet. Details of the magnet as well as all other constructional details are given in Appendix II.

The pick-up from which the amplitude of motion of the mass was determined was formed as follows:

Mounted on the end of one of the springs, along the center line of the mass, was a thin sheet of clear film negative half of which was blacked over. On one side of the film a light was mounted and on the other side was placed a small photo cell. The face of the photo cell was covered over except for a square opening at the center.



The orientation of the pieces was such that as the mass moved back and forth more or less light would reach the photo cell. The current produced by the photo cell was proportional to the amount of light received (see Figure E-1) and therefore proportional to the amplitude of motion of the mass. The current was amplified and fed to the galvanometer coils of an electromagnetic recorder. The pen of the recorder

then traced the motion of the vibrating mass. The current from the photo cell always being in the same direction, regardless of the position of the mass with respect to its rest position, resulted in a trace which was a series of positive sinusoidal pulses, each of which represents a swing of the mass past the rest position. Due to the orientation of the photo cell relative to the moving part of the pick-up and the characteristics of the photo cell and of the amplifier, the trace appears somewhat as a complete sinusoid. Actually it is not a true sine wave; the distance from the center line to the positive side only of the curve represents the motion of the mass. The type of pick-up described is considered to be satisfactory in that satisfactory results were obtained from it. A more direct method, however, would be preferable since in the type used some distortions were present. A perfect pick-up would be one which:

a. Records faithfully and continuously the motion, with due respect to sign.

b. Imposes no load whatsoever on the system.

c. Is capable of easy adjustment for orientation.

d. Is simple and cheap to operate.

The pick-up used here met fully requirements b, c and d. It met requirement "a" to a sufficient degree that satisfactory results were obtained, but no more.

For the experiments on the cylindrical conductor the mass consisted of a thin-walled copper cylinder. The cylinder was suspended by being attached on its top circumferential surface to a thin bar of stiff plastic material. The end of the bar was pivoted in a bearing which was designed to be frictionless. Also attached to the top of the cylinder and at right angles to the bar, was a length of flat spring. The other end of the spring was clamped rigidly. The object of such a suspension was to provide linear motion of the cylinder in its axial direction. The suspension was ultimately evaluated as being rather poor. It did provide the necessary rigidity,

but introduced a fairly large amount of friction and fluid (air) damping. In actual practice, with accurate workmanship, suspensions for this type of damper can and are being made wherein the motion is linear to very fine tolerances and without an undue amount of friction and fluid damping.

The magnet for this type was one which provided the desired type field. Its physical characteristics can best be seen by referring to the sketches in Appendix II.

The pick-up for tests on the cylindrical damper was essentially as described heretofore. The only difference in this case was that the moving part, instead of being directly above the moving mass, was placed on an extension of the plastic bar which held the mass. The object of the extension was to provide a magnification of the motion at the pick-up. The amplitude of motion here was considerably smaller than with the flat plate tests, being on the order of 0.06 inches, in order to preserve linearity of motioh. The degree of magnification was approximately 2.

The mass-spring system was assumed to obey the differential equation.

$$m\ddot{x}+F\dot{x}+kx=0$$

The solution of this equation is

$$x=e^{-at}(A\cos bt+B\sin bt)$$

With initial conditions that at $t=0$, $x=x_o$, $\dot{x}=0$ the solution for x can be written

$$x=\frac{x_o e^{-cw_n t}}{\sqrt{1-c^2}}\sin\left(w_n\sqrt{1-c^2}\,t+\phi\right)$$

where

$c=$ damping factor

$w=$ undamped natural circular frequency

$\phi=$ phase angle

In terms of the original parameters

$$c=\frac{F}{2\sqrt{km}}$$

$$w_n=\sqrt{\frac{k}{m}}$$

$$\tan\phi=\frac{1}{c}\sqrt{1-c^2}$$

Since c is proportional to the force acting on the system, it is this factor which, if known, can be used to examine the general equations for force developed in Section II. In order to determine c from the trace of the recorder, it is necessary to get c as a function of the maximum amplitude of vibration of the mass. To do this the expression for $\dot{x}$ above is differentiated with respect to time, set equal to zero and the result solved for c. The following formula is then obtained:

$$c = \frac{\ln \dfrac{x_o}{x_n}}{\sqrt{1.86n^2 + \left\{\ln \dfrac{x_o}{x_n}\right\}^2}}$$

where

x_o = First considered maximum amplitude measured from the rest position.

x_n = Maximum amplitude of the nth succeeding swing past the rest position, positive and negative cycles being considered.

Since the damping factor, c, is the actual quantity to be determined from each trace of the motion, it is necessary to account for not only those factors which affect force as given in the three general formulae, but also for those factors affecting c due to the type of device used. These factors are m and k. Actually k was constant throughout all tests on flat plates and again constant throughout all tests on the cylindrical conductor so that m is the only factor which was taken into account as the occasion arose.

For the description of each test which follows the detailed results and computations are listed in Appendix I.

B. Experimental tests preformed and discussion based on their results.

Test #1 -- To determine for the photocell the variation of current output with intensity of light received.

Figure B-1

The photocell was subjected to light from a standard 6 volt lamp. The amount of current output was measured with a panel type microammeter. The intensity of light received was measured with a photographic light intensity meter. Since the light meter was fixed off to one side of the photocell, the candle power readings are not those actually received by the photocell, but rather some constant fraction thereof.

The output of the photocell was determined to be linear with intensity of light received over the range tested. All further experiments retained the intensity of light within this range.

Test # 2 -- To determine the inherent or natural damping of the flat plate system.

Plate: Copper

1-5/8 inches square

d= 0.052 inches

Run # 1.

The damping coefficient c_n was determined to be 0.0135. The trace showed that the natural damping consists of the velocity or fluid type due to air friction with a very small amount of coulomb friction damping. The amount of coulomb friction damping was so small that it has been neglected. For all further runs, the natural damping was calculated for each trace individually since it changes as the plate configuration changes.

This test was made with the magnet entirely removed. For c_n in the other tests, the reversing switch supplying power to the magnet was used to remove as much residual magnetism as possible. The value c_n=0.0135 is used whenever the plate configuration for this test occurs.

Test #3 -- To determine the variation of force with field strength for a flat plate system.

Magnetic pole: Circular, 3/8 inches diameter

Plate: Copper

1-5/8 inches square

d = 0.052 inches.

Runs # 2-9

Figure E-2, E-3

It has been shown experimentally by others that the force does actually vary as the square of the field strength . This test and test #2 were made principally to verify that the experimental apparatus in this case was performing satisfactorily and that the curve traces were being interpreted correctly.

Since $F = 2c\sqrt{km}$ and k and m are constant, then the force, F, is proportional to c. The constant of proportionality is indicated on Figure E-2 as K. Figure E-3 shows that the magnetic field is proportional to the current through the windings within the range used; this constant of proportionality is indicated on Figure E-2 as K_1. The fact that the curve of KF versus $K_1 B_o^2$ does not go exactly

to the origin is probably due to the presence of slightly greater amount of natural damping than was used in the computation.

Further unrecorded tests showed that the iron core of the magnet started to saturate at about one ampere of field current.

On the original recorder traces for these and the other runs the small inked dash indicates the point where the magnetic field was turned on and damping started. Time lag for the current to build up in the coil windings are negligible.

In the data on the tests which follow and on the figures pertaining to the tests, the use of the letter K indicates, as in this test, proportionality. K appears in most cases with F (for force) signifying that the data under consideration is not force itself but some constant times force.

Test #4 -- To determine the variation of force with thickness of the conducting medium for a flat plate system.

Magnetic pole: Circular, 3/8 inches diameter

Plate: Copper

1-5/8 inches square

Current in magnetizing windings 0.8 amperes

Runs #10-12

Figure E-4

Three different thicknesses of plates were used: d=0.052, 0.031 and 0.020 inches.

In calculating the damping force acting on the plate, it is desired that all factors except the one being examined, thickness in this case, be kept constant. Since it was impossible to change the plate thickness and at the same time keep the mass and the ~~xxxxxxxx~~ conductivity of the plate constant, a correction factor for the change in mass was made. The factor consists of a multiplier for c which is the square root of the ratio of the masses, the plate of thickness 0.052 being taken as having the basic mass. The factor is then $\sqrt{\dfrac{d}{0.052}}$.

A similar procedure is followed in those tests which follow in which the mass does not stay constant.

This test shows that the force is proportional to the thickness of the conducting medium. It should be noted that this relation will not necessarily hold for very thick conductors since one of the assumptions made in Section I was that the conducting medium was thin enough that it could be represented by a plane.

Test #5a -- To determine the variation of force with plate configuration for a flat plate system.

Magnetic pole: Square 17/64 x 17/64 inches

Plate: Copper

d= 0.052 inches

Length in direction perpendicular to direction of motion 1-5/8 inches

Current in magnetizing windings 0.9 amperes

Runs #31-48 (odd)

Figure E-5, E-6

Runs were made in which the length of the plate perpendicular to the direction of motion was held constant and the length in the direction of motion varied. This procedure corresponds to a symmetrical plate termination along x as described in Section II. The length of 1-5/8 inches in the perpendicular (y) direction corresponded to an infinite length since by the analysis a y termination is always less than the equivalent x termination.

Again the mass could not be kept constant and the corrective multiplying factor for c in this case is $\sqrt{\dfrac{\text{total plate length}}{1.625}}$

With this correction made the corrected damping factor is proportional to the force and will be a function of plate configuration. Figure E-5 shows this variation. The absissa is in units of pole dimensions as used in Section II. By drawing a horizontal asymptote to the curve and relabeling the ordinate with the asymptote at unity, the ordinate then becomes equivalent to C'. Figure E-6 shows the comparison of C' as developed analytically in Section II and as found experimentally. The experimental curve is within 4% of the analytical curve for an x termination greater than 1. Practical values of x termination would probably be not less than 2 or 3.

The deviation between the two curves can probably be accounted for as follows:

For values of x less than 1.25 the experimental curve shows that less force is acting on the plate than the analytical curve indicates. The analytical curve assumes an amplitude of motion approaching zero. Actually the edge of the plate was so close to the edge of the pole face that during portions of

the motion the plate edge came under the pole face. Under these circumstances the currents in that side of the plate were almost totally interrupted during that portion of the cycle, making the average force less than it would be if the small amplitude assumption could be rigidly adhered to.

For values of x termination greater than 1.25, the experimental curve shows that there is more force acting on the plate than the analytic curve indicates. The analytic curve assumes that there are no currents in the plate above and below a certain region of the plate which is determined by the value of x termination. Actually the current density distribution in the plate must be continuous throughout the plate and there are currents where the theoretical analysis assumes there are none. These "excess" currents would account for the force being greater than predicted.

Test #5b -- To determine the variation of force with plate configuration for a flat plate system.

Magnetic pole: Square, 17/64 x 17/64 inches

Plate: Copper

$d = 0.052$ inches

Length in direction of motion - 1-5/8 inches

Current in magnetizing windings 0.9 amperes

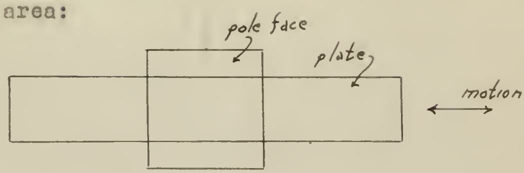
Runs #31-48 (even)

Figure E-7, E-8

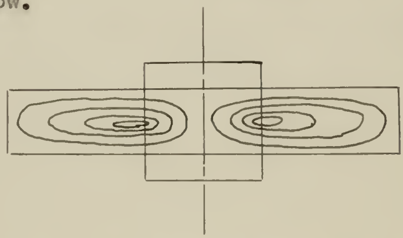
Runs for this test were made in a similar manner as for test #5a except that the plate length in the direction of motion was held constant and the length in the direction perpendicular to the motion was varied. This procedure corresponds to a symmetrical plate termination along y. Figure E-7 shows the variation of force with this plate termination. Figure E-8 shows the comparison of the experimental and analytical results. The analytical curve was drawn using the procedure described in Section II.

The difference between the two curves can probably be entirely accounted for by the failure of the current density distribution in the plate to be discontinuous, as described under test #5a. This error in the analytic assumption is shown clearly in this test.

Consider the case where the plate termination is such that the y dimension lies entirely within the pole face area:



The analytical assumption is that there will be no currents in the plate and hence no force on the plate. Actually there will be currents flowing with a density distribution of some form similar to the sketch below.



These currents, whatever their distribution, might be, will lead to a force on the plate.

These tests verify that the assumptions made in Section II are valid to within probable engineering accuracy limits.

Test #6 -- To determine the variation of force as a function of pole configuration for a flat plate system.

Magnetic pole: Rectangular of varying shape

Plate: Copper

1-5/8 inches square

$d = 0.052$ inches

Total magnetic flux $\Phi = 890$ maxwells

Runs #49-56

Figure E-9, E-10

In this test the size and shape of the plate were kept fixed and the pole face configuration varied. This variation was from a square pole face of dimensions 17/64 x 17/64 inches to a rectangle of dimensions 17/64 x 3/64 inches with the long side in the direction perpendicular to the motion of the plate. To conform to the notation of Section II, the dimension b of the pole face was constant while the dimension 2a was varied.

The total magnetic flux was kept constant as the pole face area decreased by increasing the current in the magnetizing windings. A value of

890 maxwells was maintained, current following the curve of Figure A-10.

As the pole face configuration was changed, the relative size of the plate changed. This change is accounted for by using the factor C'. Since C' had been calculated for only two pole configurations, a square and a 3 x 1 rectangle, the factor for each run made in this test was an interpolation between these two cases.

By the above consideration all factors have been accounted for except C, for which the test was run, and A, the area of the pole face. It would be desired to keep A constant leaving C as the only remaining variable, but with the apparatus used this procedure was impracticable. It could be assumed that the force does vary as A^{-1} as indicated in the formula for F and account for it in this way. An equally good assumption is that the force does vary with C as shown on Figure A-1, account for it in this way, and let the variable under test be A. This latter

procedure was the one followed principally because the resulting curve should then be a straight line through the origin rather than a curved line. Since the experimental points do plot fairly well as a straight line through the origin, it follows that the product CA^{-1} as a factor is correct.

Tests to check the variation of force with σ were not made.

Test #7a -- To determine the variation of force with plate configuration for a flat plate system.

Magnetic pole: Circular, 3/8 inch diameter

Plate: Copper

d=0.052 inches

Length in direction perpendicular to direction of motion 1-5/8 inches

Current in magnetizing windings 0.8 amperes

Runs #13-21

Figure E-11

This test was conducted in a manner similar to test #5a and for the same purpose. Again the multiplier for c to take care of the effect of change of mass of the plate is $\sqrt{\dfrac{\text{total plate length}}{1.625}}$.

Figure E-11 shows the results of the test. By drawing the horizontal asymptote to the curve and giving the ordinate at this asymptote, a value of 1 the values of C' are established.

Since a corresponding analytical curve has not been developed, a comparison can not be made. As the analogous curves for the rectangular pole face

are in quite close correspondence, it can be assumed that Figure E-11 is correct to within the same limits.

Test #7b -- To determine the variation of force with plate configuration for a flat plate system.

Magnetic pole: Circular, 3/8 inch diameter

Plate: Copper

d=0.052 inches

Length in direction of motion 1-5/8 inches

Current in magnetizing windings 0.8 amperes

Runs #22-30

Figure E-12, E-13, E-14

This test was conducted in a manner similar to test #5b and for the same purpose.

Figure E-12 shows the results of the test.

Using the data from Figure E-11 and E-12, the curves of constant current density can now be sketched. Figure E-13 shows these curves for the first quadrant of the infinite plate. These curves have been sketched, not plotted, from the following considerations:

1. Their general shape should be similar to Figure A-2 and A-6.

2. They should have vertical tangents along a diameter of the pole face in the direction of motion of the plate. The values for these points have been taken from Figure E-11.

3. They should have horizontal tangents corresponding to the curve of Figure E-12.

The method of sketching was as follows:

For a representative curve consider the one labeled "a". From Figure E-11 an x termination of 3 units results in a value of C' of 0.775. From Figure E-12 the value of y termination which has an equivalent C' of 0.775 is found to occur at 1.75. Thus curve "a" of Figure E-13 has a vertical tangent at x=3 units and a horizontal tangent at y=1.75 units. From these two values and a knowledge of its general shape curve "a" was sketched. The curves of Figure E-13 have not been labeled as to relative intensity except to note that their relative intensity increases toward the origin of coordinates.

Figure E-14 shows the values of the total relative dimensions for a flat plate such that the corresponding x and y terminations will be tangent to the same curve of constant current density. It was drawn from values taken off Figure E-13 and corresponds to Figures A-5 and A-9.

Test 8 -- Analysis to verify the factor for pole face configuration for a flat plate system employing a circular magnetic pole face.

The analysis of Section II showed that for a flat plate system employing a rectangular pole face, there is a factor C in the equation for force which is dependent on the ratio of the lengths of the sides of the pole face; and that in the case of a circular pole face the corresponding factor has a constant value of 0.5 In order to verify this value of 0.5 let the force equation be written as

$$F = \Phi^2 \sigma v dA^{-1} C' K'$$

where K' is now an unknown factor. This equation corresponds dimensionally to that for the force on a system employing a rectangular pole face:

$$F = \Phi^2 \sigma v dA^{-1} C' C$$

if K' is dimensionless. Also, since the factor C expresses the ratio of the parameters defining the rectangular pole face and the circle is defined by only one parameter, it can be concluded that K' is not only a dimensionless, but also a constant, factor. The two equations can now be divided one by the other to give:

$$\frac{F_C}{F_R} = \frac{\Phi_C^2 C_C' A_R K'}{\Phi_R^2 C_R C_R' A_C}$$

The subscripts c indicate values for the system having a circular pole face, subscripts R indicate values for the system having a rectangular pole face.

F_c and F_R are force per unit velocity. The factors σ and d have been cancelled out since they are the same for the experimental tests on each of the two systems.

Now using Run #49 and Run #2, the following values can be listed:

$\Phi_R = 890$ maxwells

$\Phi_R^2 = 79.21 \cdot 10^4$

$A_R = 0.4545 \text{cm}^2$

$C_R = 0.5$

$C_R' = 0.935$

$\Phi_c = 1300$ maxwells (from Figure [illegible]-3 using 5% leakage flux)

$\Phi_c^2 = 169 \cdot 10^4$

$A_c = 0.713 \text{cm}^2$

$C_c' = 0.91$

Substituting these values in the equation above:

$$\frac{F_c}{F_R} = 2.645K'$$

or

$$K' = 0.378 \frac{F_c}{F_R}$$

Since the masses of the plates and the spring constants were the same for Runs #2 and #49:

$$\frac{f_c}{f_R} = \frac{c_c \text{ (Run \#2)}}{c_c \text{ (Run \#49)}} = \frac{0.1445}{0.1185} = 1.22$$

Thus

$$K' = 0.378 \cdot 1.22 = 0.461$$

The analytical value of K' is 0.5 which is verified within experimental accuracy by the value 0.461 found above. The major inaccuracy of the above computation is probably in the values of Φ.

Test #9 -- To determine for a cylindrical conductor system the variation of force with cylinder length.

Magnetic pole: b= 0.125 inches

Cylinder: Copper

Diameter 0.449 inches (mean)

d=0.035 inches

Current in magnetizing windings 0.16 amperes

Runs #57-71

Figure E-15, E-16

Runs were made in which the length of the conducting cylinder was varied. The cylinder was oriented in the magnetic field as nearly symmetrically as possible so that as much of the cylinder was above the upper pole surface as was below the lower pole surface when the cylinder was in the rest position.

The only factor which was varied (beside the cylinder length) was the mass. A multiplier for c was found in a similar manner as described heretofore except that in this case a slightly more complicated procedure was necessary due to the fact that the mass of the support members was not negligible with respect to the mass of the cylinder. The details of arriving at the factor used are given in Appendix I.

The results of this test as shown on Figure E-15 show that the force acting on the cylinder is independent of the length of the cylinder outside of the region of flux. The analytical formula for force as developed in Section II indicates this independence by the absence of the factor C' and the experimental test substantiates the analysis.

In setting up the equipment for this test the cylinder was first held on its suspension in such a manner that the entire top of the cylinder was covered by a thin sheet of fiber material. It was noticed that under this condition a very high value of natural damping occured. The reason for the high natural damping was found to be due to the fact that the relatively small clearances between the inner magnet pole and the cylinder walls produced a piston-cylinder effect. When the support was changed so that only a small part of the top of the cylinder was covered, the natural damping decreased to an acceptable value. Thus in order to

obtain the minimum amount of fluid damping, which in a magnetic damping system is undesirable, the end of the cylinder should be as clear as possible.

Test #10 -- To determine for a cylindrical conductor system the variation of force with pole face configuration.

Cylinder: Copper

Length 0.672 inches

d=0.015 inches

Diameter 0.449 inches (mean)

Total magnetic flux Φ =1223 maxwells

Runs #72-86

Figure E-17, E-18

In this test the size and shape of the conducting cylinder were kept fixed and the pole face configuration varied. This variation was accomplished by varying b, the pole distance as defined in Section II from 0.312 inches to 0.125 inches.

The total magnetic flux was kept constant as the mean pole face area decreased by increasing the current in the magnetizing windings. A value of Φ =1223 maxwells was maintained, the current following the curve of Figure E-18. Due to the fact that the mean area of the pole face changed as the configuration was changed, a procedure similar to

that described under test #6 was followed. Figure E-17 shows that the product CA^{-1} as a factor is correct.

APPENDIX I

TEST DETAILS

Test #1

To determine for the photocell the variation of current output with intensity of light received.

I	Candle Power
10.5	24
8	17
6.5	15
4	10
2	4

I = current output in microamperes

Test #2

To determine the inherent or natural damping of the flat plate system.

$c_n = 0.0135$

Notation for further test details:

c = damping factor as determined from individual trace of motion.

c_n = natural damping factor as determined from individual trace of motion.

c_c = corrected damping factor which is that due to the magnetic damping effect alone. $c_c = c - c_n$

Test #3

To determine the variaion of force with field strength for a flat plate system.

Run	I	$I^2 = K_1 B_0^2$	c	c_n	$c_c = KF$
2	0.8	0.64	0.158	0.0135	0.1445
3	0.7	0.49	0.1218	"	0.1083
4	0.6	0.36	0.0935	"	0.0818
5	0.5	0.25	0.0785	"	0.065
6	0.4	0.16	0.0521	"	0.0386
7	0.3	0.09	0.03985	"	0.02635
8	0.2	0.04	0.02565	"	0.01215
9	0.14	0.02	0.0214	"	0.0079

I= current in magnetizing windings in amperes.

Test #4

To determine the variation of force with thickness for a flat plate system.

Run	d	c	cn	c_c	M	$c_cM = KF$
10	0.052	0.158	0.0135	0.1445	1	0.1445
11	0.031	0.129	0.00863	0.1204	0.772	0.093
12	0.020	0.1007	0.00825	0.09245	0.62	0.0573

d = plate thickness in inches

M = correction factor for mass effect

$$= \sqrt{\frac{d}{0.052}}$$

The weight of the support members has not been taken into account in calculating the relative change in mass (M) since the weight of the lightest plate tested was more than twenty times the weight of the support members.

Total weight of support members: pickup section, fiber support arm and springs including portions of springs held in clamps = 0.3525 grams.

Weight of lightest plate tested (d= 0.020) = 8.35 grams.

Test #5a

To determine the variation of force with plate configuration for a flat plate system.

Run	L	c_n	c	c_c	M	$c_c M = KF$	x termination
31	1-5/8	0.0135	0.1735	0.16	1.0	0.16	3.06
33	1-3/8	0.013	0.173	0.16	0.92	0.1471	2.59
35	1-1/8	0.009	0.17	0.161	0.8325	0.134	2.115
37	7/8	0.006	0.1956	0.1896	0.735	0.139	1.65
39	3/4	0.0051	0.188	0.1829	0.68	0.1243	1.41
41	5/8	0.005	0.176	0.171	0.62	0.1061	1.176
43	1/2	0.0032	0.146	0.1428	0.555	0.0793	0.941
45	3/8	0.0026	0.0663	0.0637	0.48	0.0306	0.707
47	1/4	0.002	0.01614	0.014	0.392	0.00549	0.4705

L= Plate length in the direction of motion in inches

M= Correction factor for mass effect

$$= \sqrt{\frac{L}{1.625}}$$

Weight of support members has again not been taken into account.

Weight of lightest plate tested (1-5/8 x 1/4, d= 0.052)= 3.340 grams.

x termination= L converted to units of pole dimensions.

Test #5b

To determine the variation of force with plate configuration for a flat plate system.

Run	L	c_n	c	c_o	M	c_oM=KF	y termination
32	1-5/8	0.0135	0.1735	0.16	1.0	0.16	3.06
34	1-3/8	0.0079	0.178	0.17	0.92	0.1563	2.59
36	1-1/8	0.0072	0.1932	0.186	0.8325	0.155	2.115
38	7/8	0.0065	0.221	0.2145	0.735	0.1578	1.65
40	3/4	0.006	0.21	0.204	0.68	0.1388	1.41
42	5/8	0.0055	0.2155	0.21	0.62	0.1302	1.176
44	1/2	0.0034	0.216	0.2126	0.555	0.118	0.941
46	3/8	0.0029	0.207	0.2041	0.48	0.0984	0.707
48	1/4	0.0023	0.1202	0.1179	0.392	0.0462	0.4705

L =Plate length in direction perpendicular to direction of motion in inches.

M =Correction factor for mass effect.

$$= \sqrt{\frac{L}{1.625}}$$

y termination= L converted to units of pole dimensions.

Test #6

To determine the variation of force as a function of pole configuration for a flat plate system.

Run	2a	b/2a	I	cn	c	c_c	R	P	$RPc_c = KF$	$\frac{1}{2a} = \frac{X'}{\lambda}$
49	17	1.0	0.82	0.0135	0.132	0.1185	1	1	0.1185	0.0588
50	15	1.132	0.92	"	0.167	0.1535	0.991	0.95	0.1447	0.0666
51	13	1.308	1.05	"	0.145	0.1315	0.981	0.905	0.1169	0.0769
52	11	1.545	1.2	"	0.180	0.1665	0.971	0.848	0.137	0.091
53	9	1.89	1.5	"	0.213	0.200	0.961	0.784	0.151	0.111
54	7	2.428	1.95	"	0.2755	0.262	0.952	0.726	0.182	0.1429
55	5	3.4	2.68	"	0.42	0.4065	0.943	0.675	0.259	0.20
56	3	5.666	4.45	"	0.745	0.732	0.935	0.615	0.428	0.333

2a= Pole length in the direction of the motion of the mass in 64th inches.

b= Pole length in the perpendicular direction in 64th inches.

I= Current in magnetizing windings in amperes.

R= Correction factor for change in relative configuration of the plate.

P= Correction factor for change in pole face configuration.

(See next page for computation of R and P.)

Computation of factors R and P, test 6.

Run	C'	$\frac{0.935}{C'} = R$	C	$\frac{0.5}{C} = P$
49	0.935	1.0	0.5	1.0
50	0.944	0.991	0.526	0.95
51	0.953	0.981	0.553	0.905
52	0.963	0.971	0.59	0.848
53	0.973	0.961	0.638	0.784
54	0.982	0.952	0.689	0.726
55	0.991	0.943	0.741	0.675
56	1.0	0.935	0.812	0.615

C' is the factor for plate configuration interpolated between data for square pole face, Figure A-4, and for a 3x1 rectangular pole face, Figure A-8.

C is the factor for change in pole face configuration, Figure A-1.

Test 7a

To determine the variation of force with plate configuration for a flat plate system.

Run	L	c_n	c	c_c	M	$c_c M = KF$	x termination
13	1-5/8	0.0135	0.158	0.1445	1	0.1445	4.33
14	1-3/8	0.013	0.165	0.152	0.92	0.14	3.67
15	1-1/8	0.009	0.159	0.15	0.8325	0.125	3.0
16	7/8	0.006	0.1374	0.1314	0.735	0.0966	2.335
17	3/4	0.0051	0.112	0.107	0.68	0.0727	2.0
18	5/8	0.005	0.1168	0.112	0.62	0.0695	1.667
19	1/2	0.0032	0.0935	0.09	0.555	0.05	1.332
20	3/8	0.0026	0.022	0.0194	0.48	0.00932	1.0
21	1/4	0.002	0.01	0.008	0.392	0.00314	0.822

L = Plate length in the direction of motion in inches

M = Correction factor for mass effect

$$= \sqrt{\frac{L}{1.625}}$$

x termination = L converted to units of pole dimensions

Test #7b

To determine the variation of force with plate configuration for a flat plate system.

Run	L	c_n	c	c_c	M	$c_c M=KF$	y termination
22	1-5/8	0.0135	0.158	0.1445	1	0.1445	4.33
23	1-3/8	0.0079	0.1456	0.1377	0.92	0.1266	3.67
24	1-1/8	0.0072	0.151	0.1438	0.8325	0.1197	3.0
25	7/8	0.0065	0.159	0.1525	0.735	0.1121	2.335
26	3/4	0.006	0.1702	0.1642	0.68	0.1118	2.0
27	5/8	0.0055	0.174	0.1685	0.62	0.1045	1.667
28	1/2	0.0034	0.153	0.1496	0.555	0.083	1.332
29	3/8	0.0029	0.1212	0.1183	0.48	0.0568	1.0
30	1/4	0.0023	0.0623	0.06	0.392	0.0235	0.822

L=Plate length in direction perpendicular to direction of motion in inches

M=Correction factor for mass effect

$$=\sqrt{\frac{L}{1.625}}$$

y termination= L converted to units of pole dimensions

Test 9

To determine for a cylindrical conductor system the variation of force with cylinder length.

Run	I	l	c_n	c	c_c	M	$c_c M = KF$
57	--	0.665	0.004	--	--	--	--
58	0.0	0.665	0.023	--	--	--	--
59	0.16	0.665	0.023	0.0962	0.0732	1.0	0.0732
60	0.0	0.625	0.02275	--	--	--	--
61	0.16	0.625	0.02275	0.0914	0.06865	0.972	0.0667
62	0.0	0.562	0.0191	--	--	--	--
63	0.16	0.562	0.0191	0.0898	0.0707	0.925	0.0654
64	0.0	0.500	0.0186	--	--	--	--
65	0.16	0.500	0.0186	0.095	0.0764	0.88	0.0672
66	0.0	0.445	0.0174	--	--	--	--
67	0.16	0.445	0.0174	0.0983	0.0809	0.821	0.0665
68	0.0	0.386	0.0165	--	--	--	--
69	0.16	0.386	0.0165	0.103	0.0865	0.784	0.0678
70	0.0	0.320	0.01	--	--	--	--
71	0.16	0.320	0.01	0.1041	0.0941	0.772	0.068

I = Current in magnetizing windings in amperes

l = Cylinder length in inches

M = Correction factor for mass effect (See next page)

Note: Run 57 was made with the magnet entirely removed to see what effect this condition would have

on the natural damping of the system. The figures on c_n of run 57 and 58 indicate this effect.

Computation of factor M, test #9.

Total weight of all support members including the portion of the spring clamped down in its support = 0.9415 grams.

Weight of shortest cylinder tested = 2.272 grams.

Effective weight of supporting members = 0.4 grams. (Estimated)

Run	W_c	W_s	W_t	$\frac{W_t}{5.12}$	$\sqrt{\frac{W_t}{5.12}} = M$
59	4.72	0.4	5.12	1.0	1.0
61	4.44	"	4.84	0.945	0.972
63	3.99	"	4.39	0.856	0.925
65	3.55	"	3.95	0.7715	0.88
67	3.16	"	3.46	0.6755	0.821
69	2.74	"	3.14	0.613	0.784
71	2.272	"	2.672	0.521	0.722

W_c = Cylinder weight in grams.

W_s = Support weight in grams.

$W_t = W_c + W_s$

Test #10

To determine for a cylindrical conductor system the variation of force with pole face configuration.

Run	I	b	$\frac{b}{D}$	c_n	c	c_c	F	$Pc=\frac{KF}{c}$	$\frac{1}{b}=$
72	--	--	--	0.00156	--	--	--	--	--
73	0.0	0.312	--	0.02825	--	--	--	--	--
74	0.132	0.312	0.695	0.02825	0.060	0.03175	1.0	0.03175	3.
75	0.0	0.257	--	0.0256	--	--	--	--	--
76	0.18	0.257	0.573	0.0256	0.120	0.0944	0.915	0.0864	3.
77	0.0	0.210	--	0.023	--	--	--	--	--
78	0.20	0.210	0.468	0.023	0.081	0.058	0.842	0.0488	4.
79	0.0	0.187	--	0.0177	--	--	--	--	--
80	0.216	0.187	0.417	0.0177	0.0865	0.0688	0.78	0.0536	5.
81	0.0	0.164	--	0.012	--	--	--	--	--
82	0.246	0.164	0.365	0.012	0.0913	0.0793	0.746	0.0591	6.
83	0.0	0.156	--	0.0109	--	--	--	--	--
84	0.256	0.156	0.348	0.0109	0.0965	0.0856	0.735	0.063	6.
85	0.0	0.125	--	0.01027	--	--	--	--	--
86	0.32	0.125	0.2785	0.01027	0.120	0.1097	0.687	0.0754	8.

I = Current in magnetizing windings in amperes.

b = Pole distance in inches.

D = Mean diameter of cylinder in inches.

Note: For run #72 see note under test #9 for run #57.

Computation for factor P, test #10.

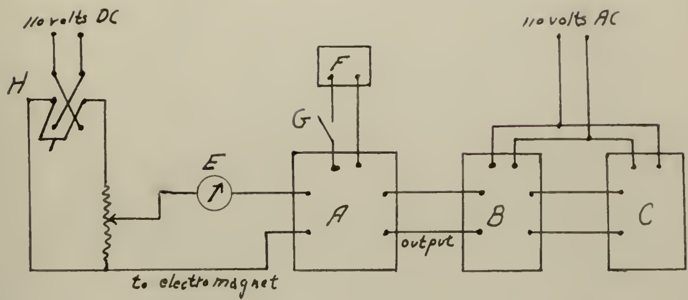
Run	C	$\frac{0.522}{C} = P$
74	0.522	1.0
76	0.57	0.915
78	0.62	0.842
80	0.67	0.78
82	0.70	0.746
84	0.71	0.735
86	0.76	0.687

C is the factor for change in pole face configuration, Figure A-12.

APPENDIX II

CONSTRUCTIONAL DETAILS

General lay-out



A. Main Unit consisting of:

1. Vibrating mass under test
2. Electromagnet
3. Photocell
4. 6 volt lamp

B. Amplifier: Brush Development Co. Model BL-902

C. Recorder: Brush Development Co. Model BL-202

D. Slide wire potentiometer : 717ohms

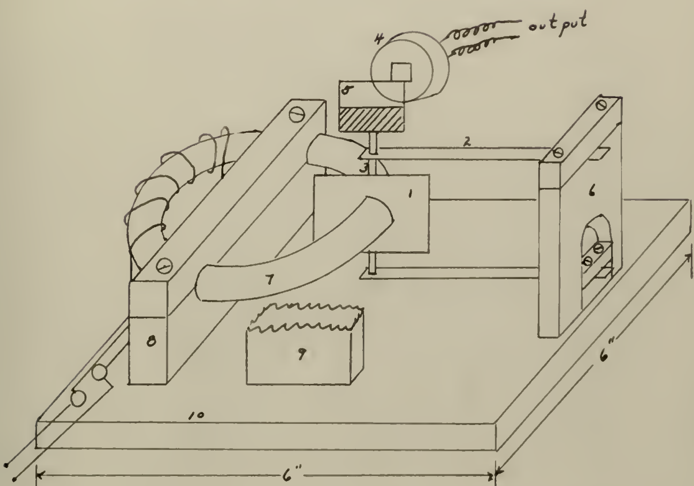
E. DC ammeter : 0.1. ampere

F. 6 volt dry cell

G. SPST switch

H. DPDT switch (reversing)

Schematic drawing of the main unit set up for flat plate tests:



1. Vibrating mass (copper plate)
2. Flat bar springs
3. Support for plate
4. Photocell
5. Moving member of pick-up.

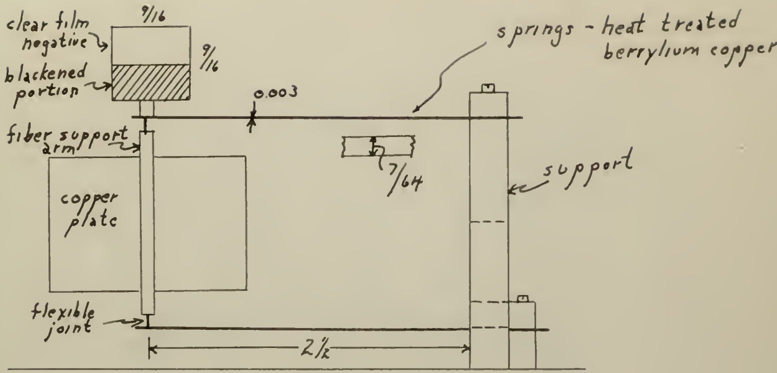
6. Support for springs
7. Electromagnet
8. Support for electromagnet
9. Support for 6 volt lamp
10. Base plate

Not shown is the lamp itself, a 6 volt bulb surrounded by an aluminum cylinder to afford a degree of collimation. The light from the lamp shines directly through the moving member of the pick-up into the face of the photocell.

Also not shown on the sketch is the support for the photocell. This support as well as those for the lamp and the springs are all adjustable to permit alignment.

The sketch shows the torus magnet in place. For tests on the cylindrical conductor the torus magnet was removed and the cylindrical magnet installed.

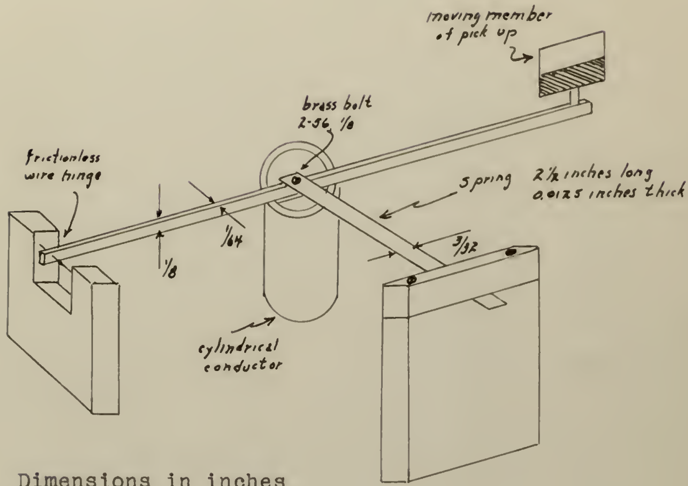
Moving system for flat plate tests:



Dimensions in inches

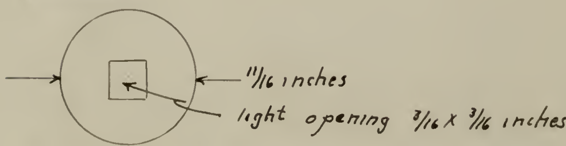
The various specimens of copper plate were cemented to the fiber support arm with commercial Ducocement.

Moving system for cylindrical conductor tests:



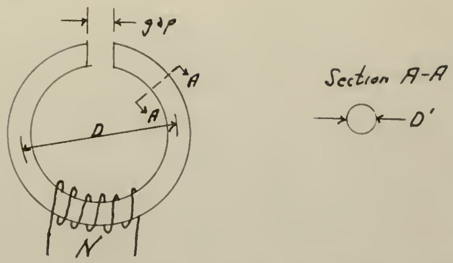
Dimensions in inches

Photocell



No data on the photocell characteristics, type, etc. is available except as shown on Figure [illegible]-1.

Torus magnet



Core aterial cold rolled steel, exact type unknown.

D= 7.3 cm

D'= 0.9525 cm

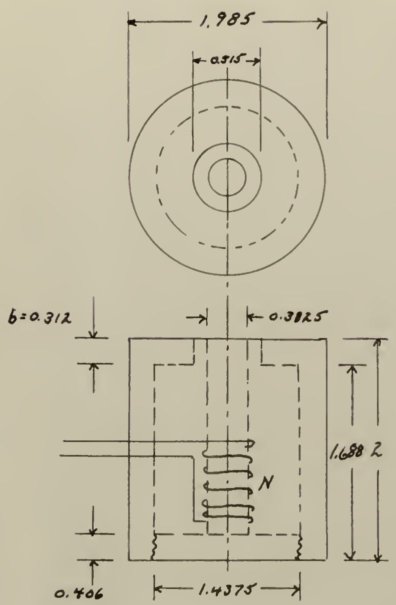
Gap =0.238 cm

N= 572 turns AWG#22 cotten covered copper magnet wire

Figure E-3 was drawn from computation for this magnet using the data for Ordinary Low Carbon Steel in Handbook of Engineering Fundamentals, Eshbach, page 11-103.

For tests involving square and rectangular pole faces, the poles of this magnet were filed down to the specified size from a taper starting about ¼ inch back of the pole face.

Cylindrical type magnet



dimensions in inches

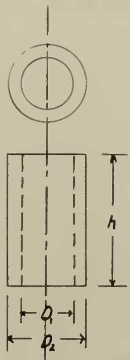
Core material cold rolled steel, exact type unknown.

N = 480 turns A.G. 25 enameled copper magnet wire

Figure E-17 was drawn using the same data as for the torus magnet.

For tests involving changes in the pole distance, b, the entire top surface of the magnet, including the center pole, was turned down to give the specified value of b.

Specifications for the conducting cylinders.



Unit	h	D_1	D_2	d
1	0.665	0.484	0.414	0.035
2	0.672	0.464	0.434	0.015

dimensions in inches

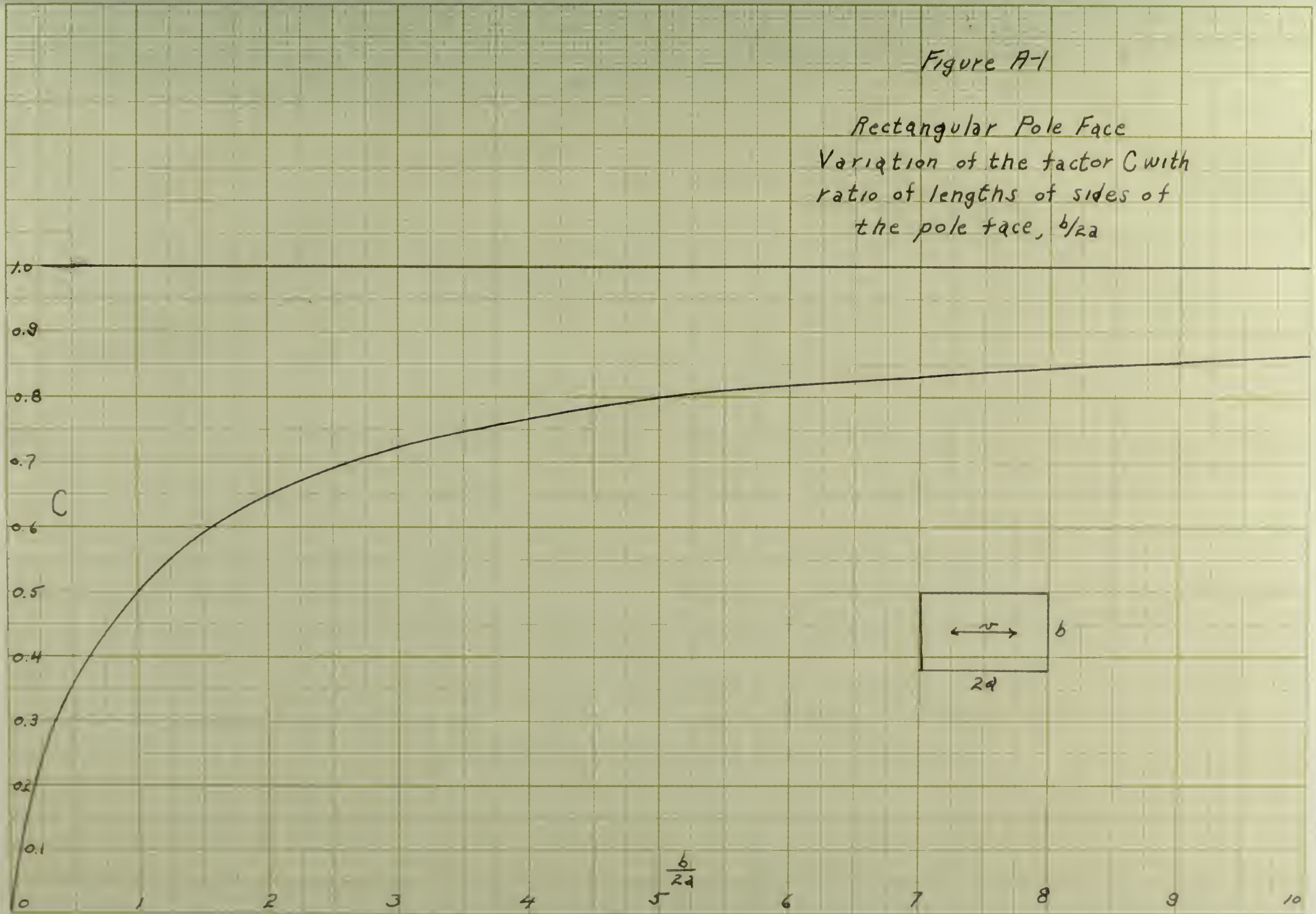


Figure A-1
Rectangular Pole Face
Variation of the factor C with
ratio of lengths of sides of
the pole face, b/2a
C
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
1
2
3
4
5
6
7
8
9
10
b/2a
b
2a

Figure 17

Rectangular Pole Face
Variation of the factor C with
ratio of lengths of sides of
the pole face a/2a

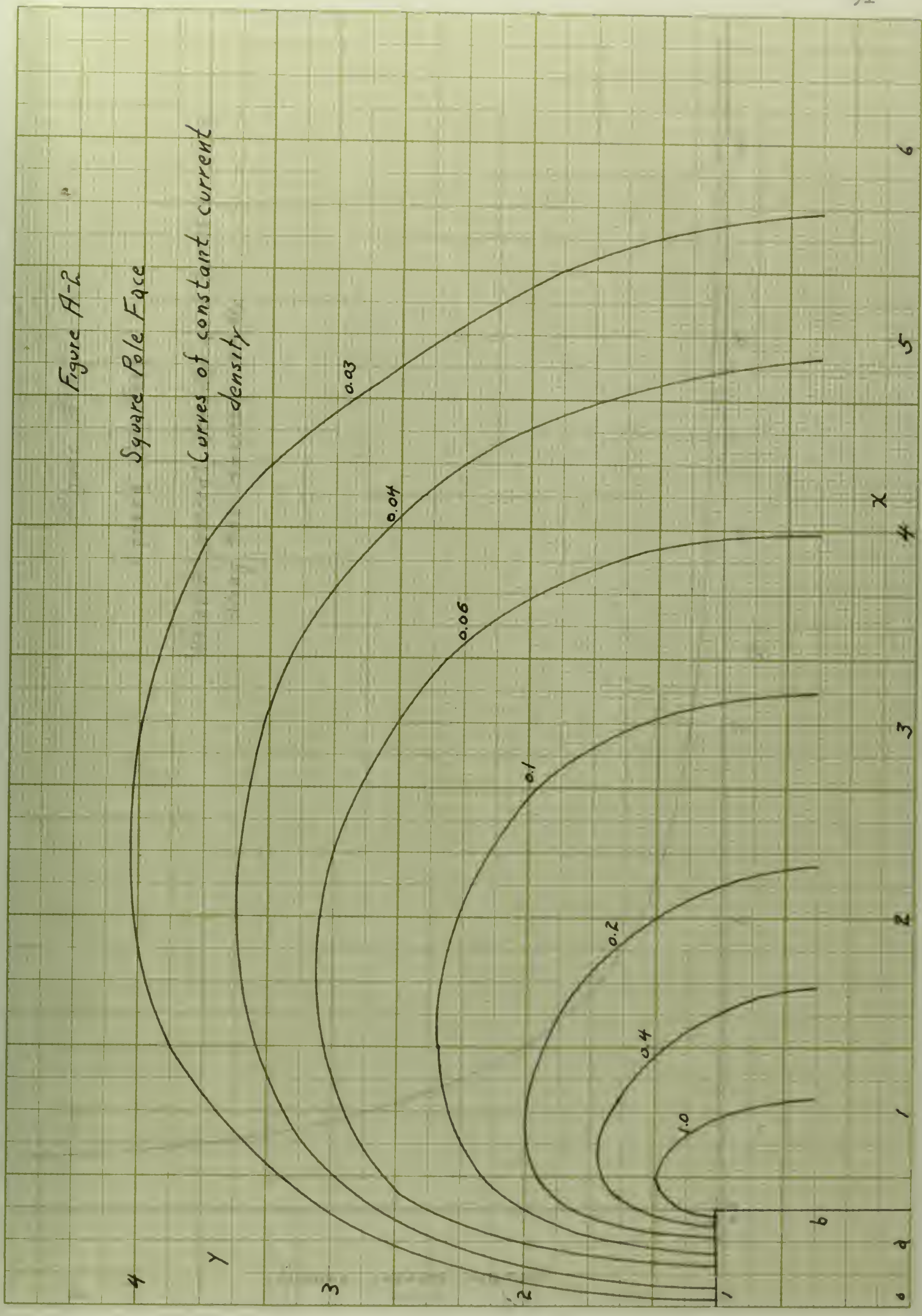
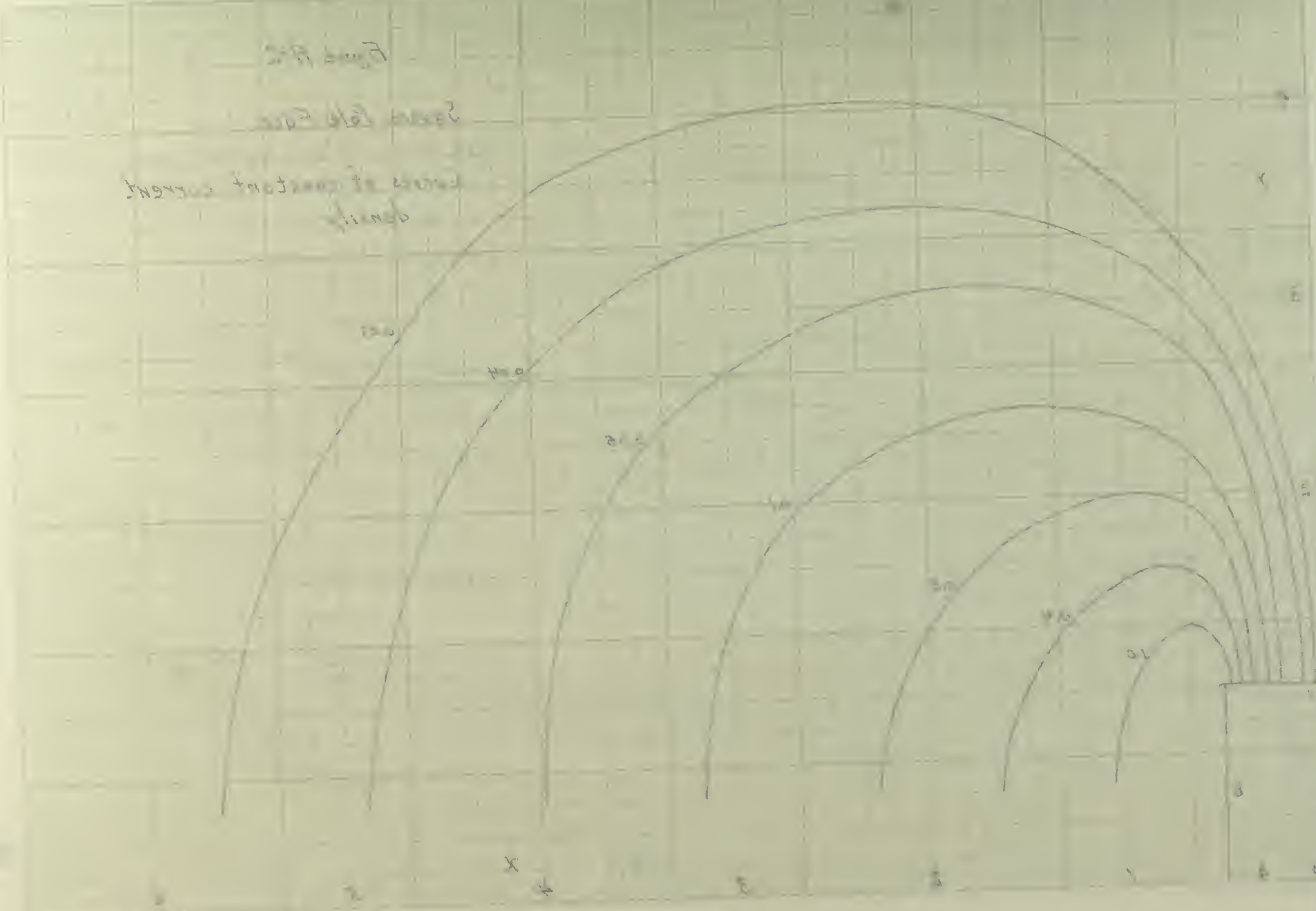


Figure A-2
Square Pole Face
Curves of constant current density



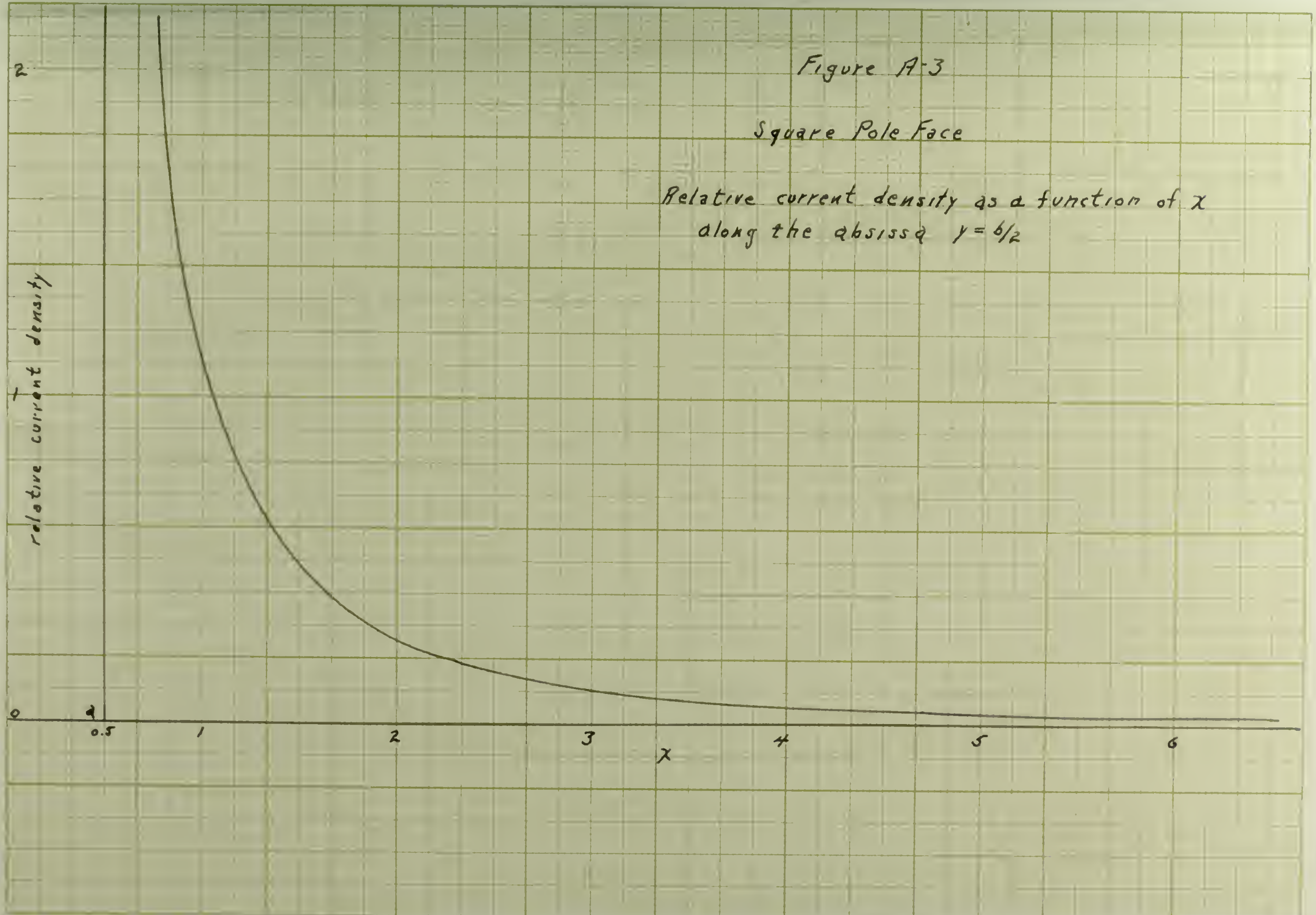


Figure A-3

Square Pole Face

Relative current density as a function of x along the absissa $y = b/2$

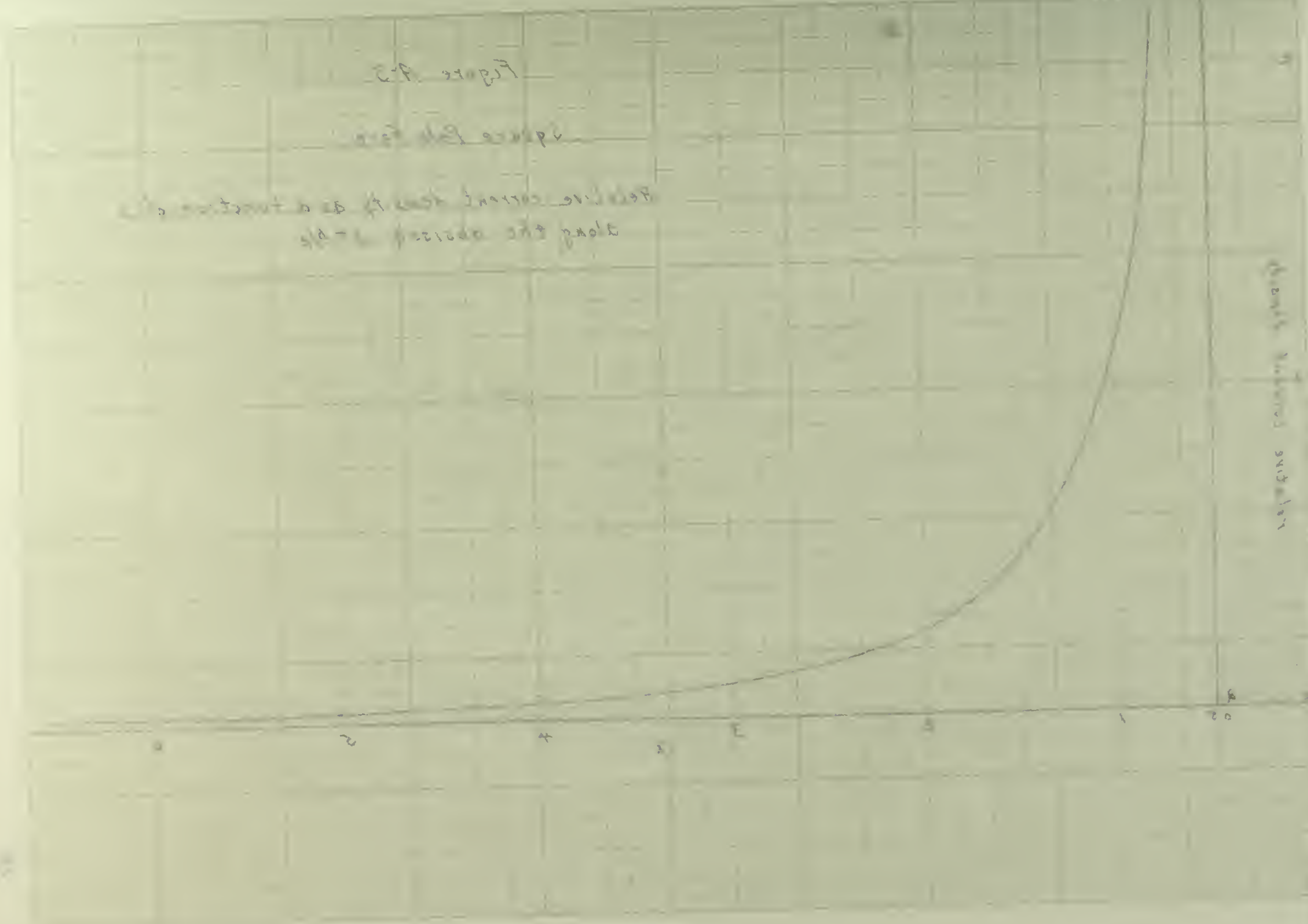
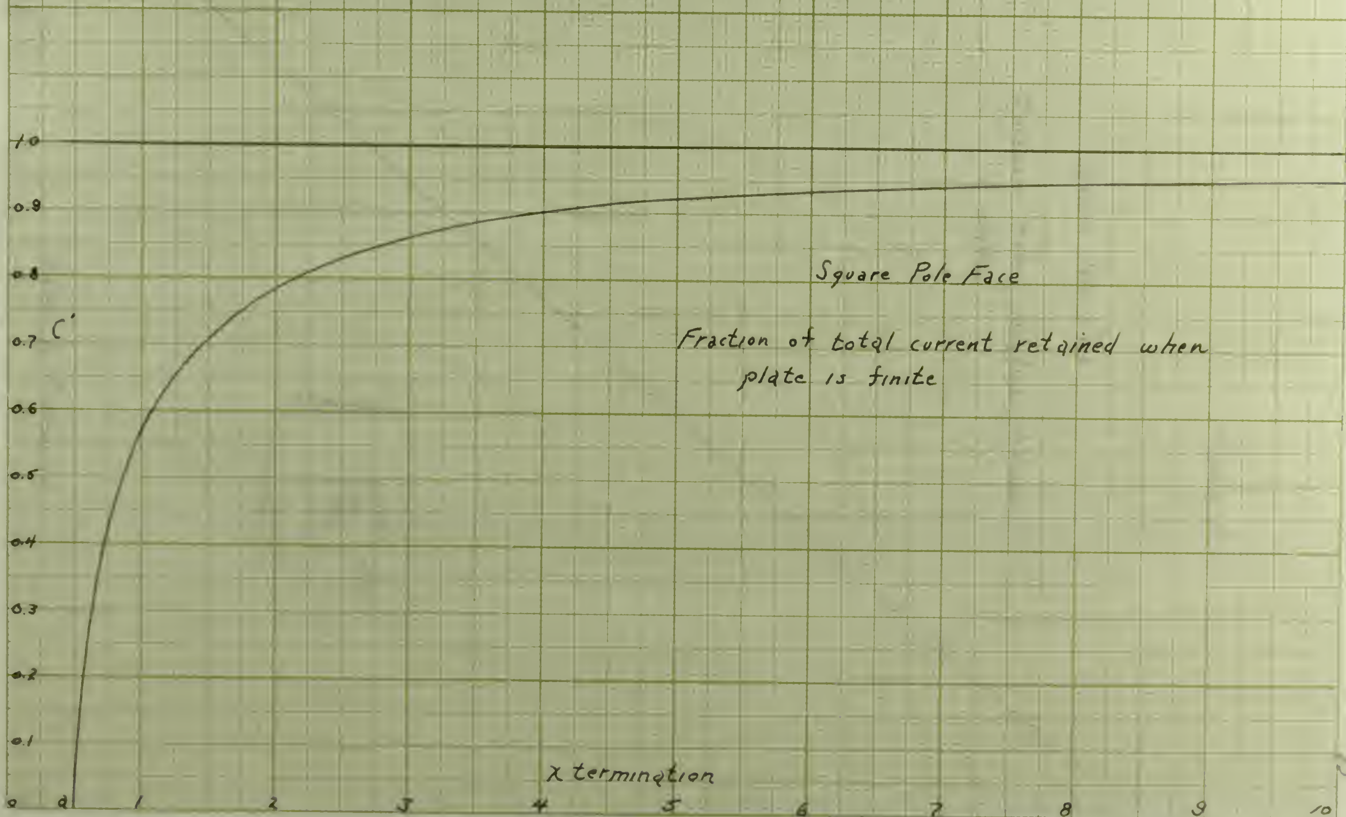
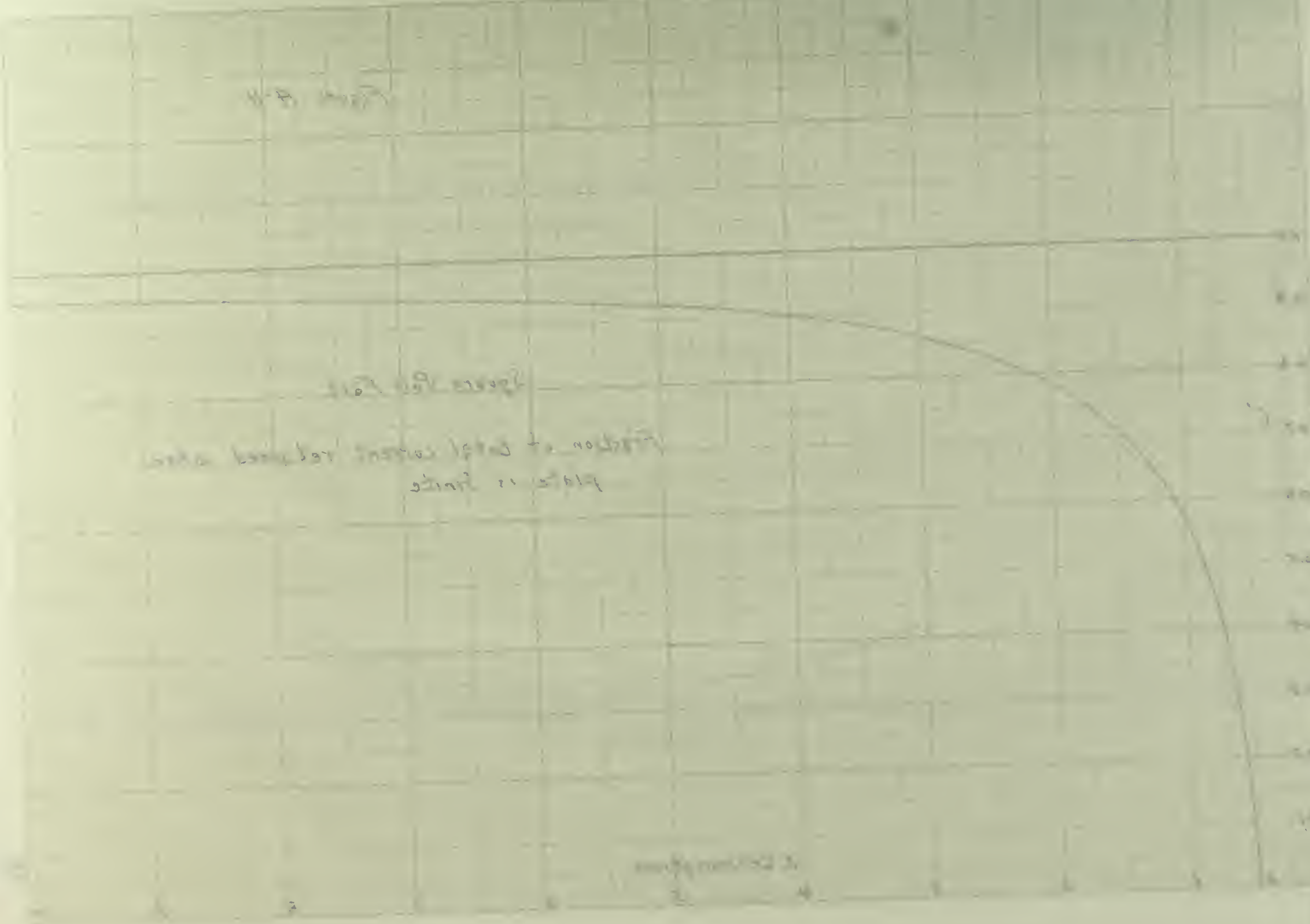


Figure 7-3
Square Pole Face
Relative current density as a function of x
along the abscissa y - b/2
Relative current density

Figure A-4


Square Pole Face
Fraction of total current retained when plate is finite
C'
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
a
1
2
3
4
5
6
7
8
9
10
x termination



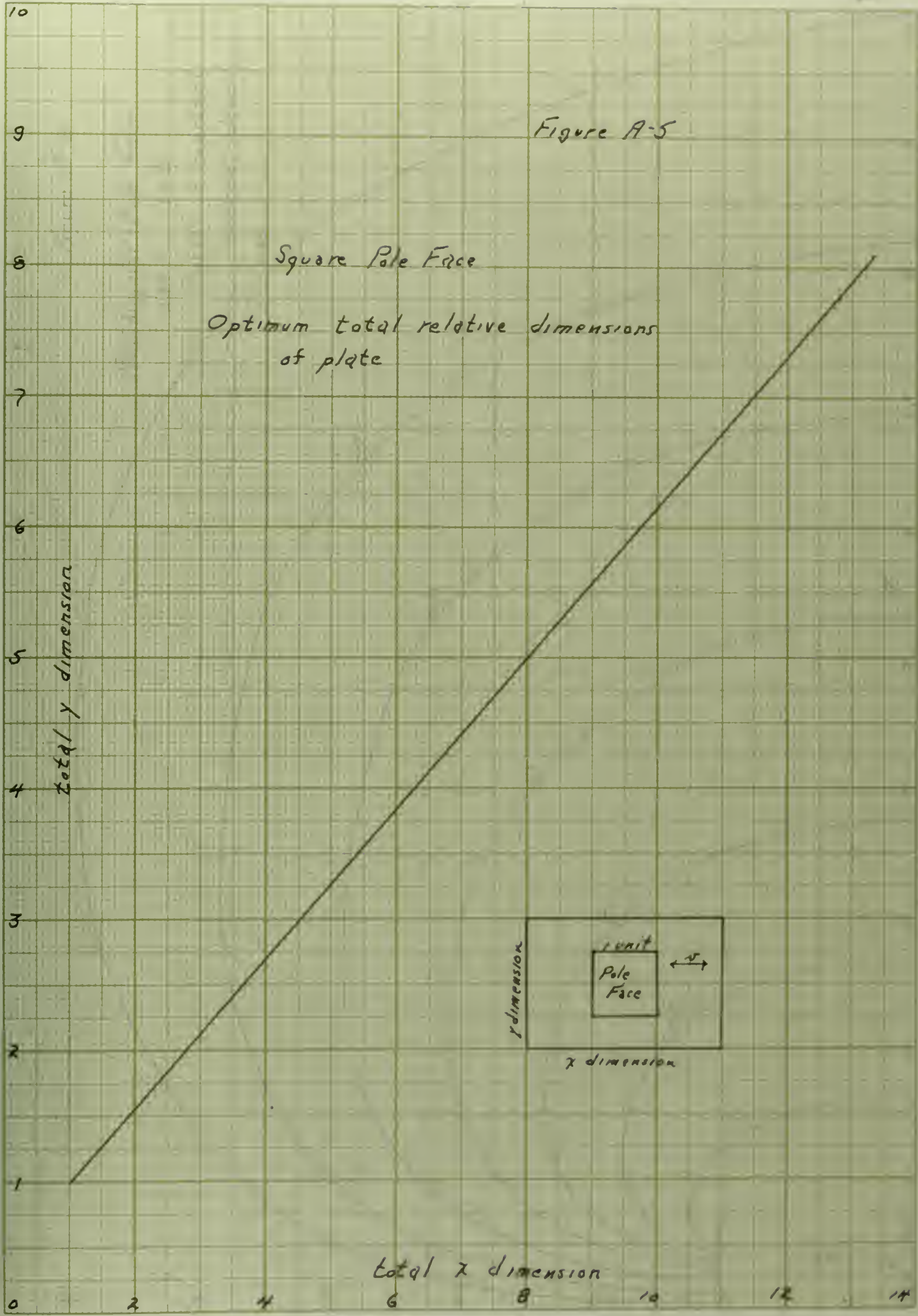


Figure A-5
Square Pole Face
Optimum total relative dimensions of plate
total y dimension
total x dimension
y dimension
x dimension
1 unit
Pole Face
v
0
1
2
3
4
5
6
7
8
9
10
0
2
4
6
8
10
12
14

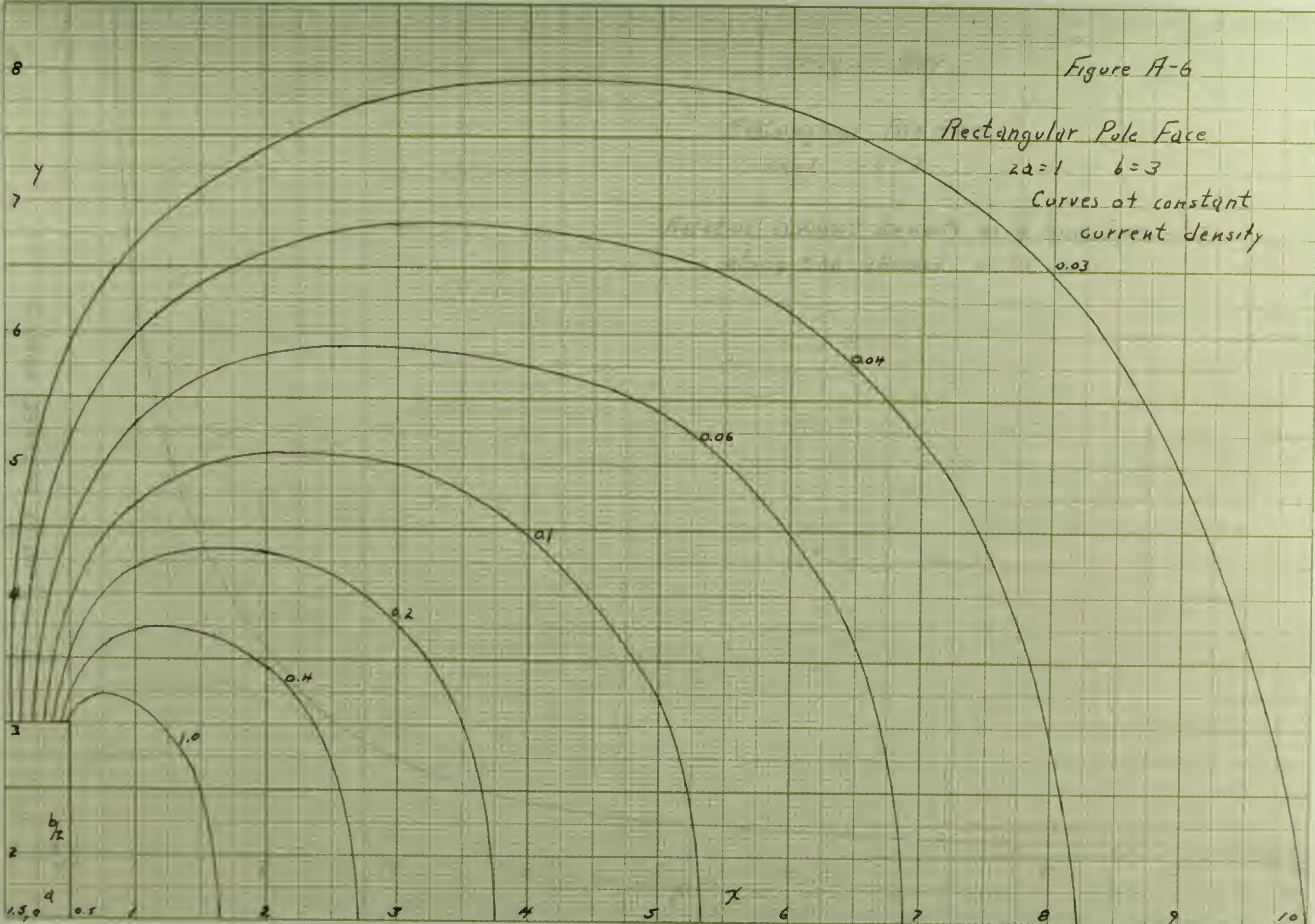
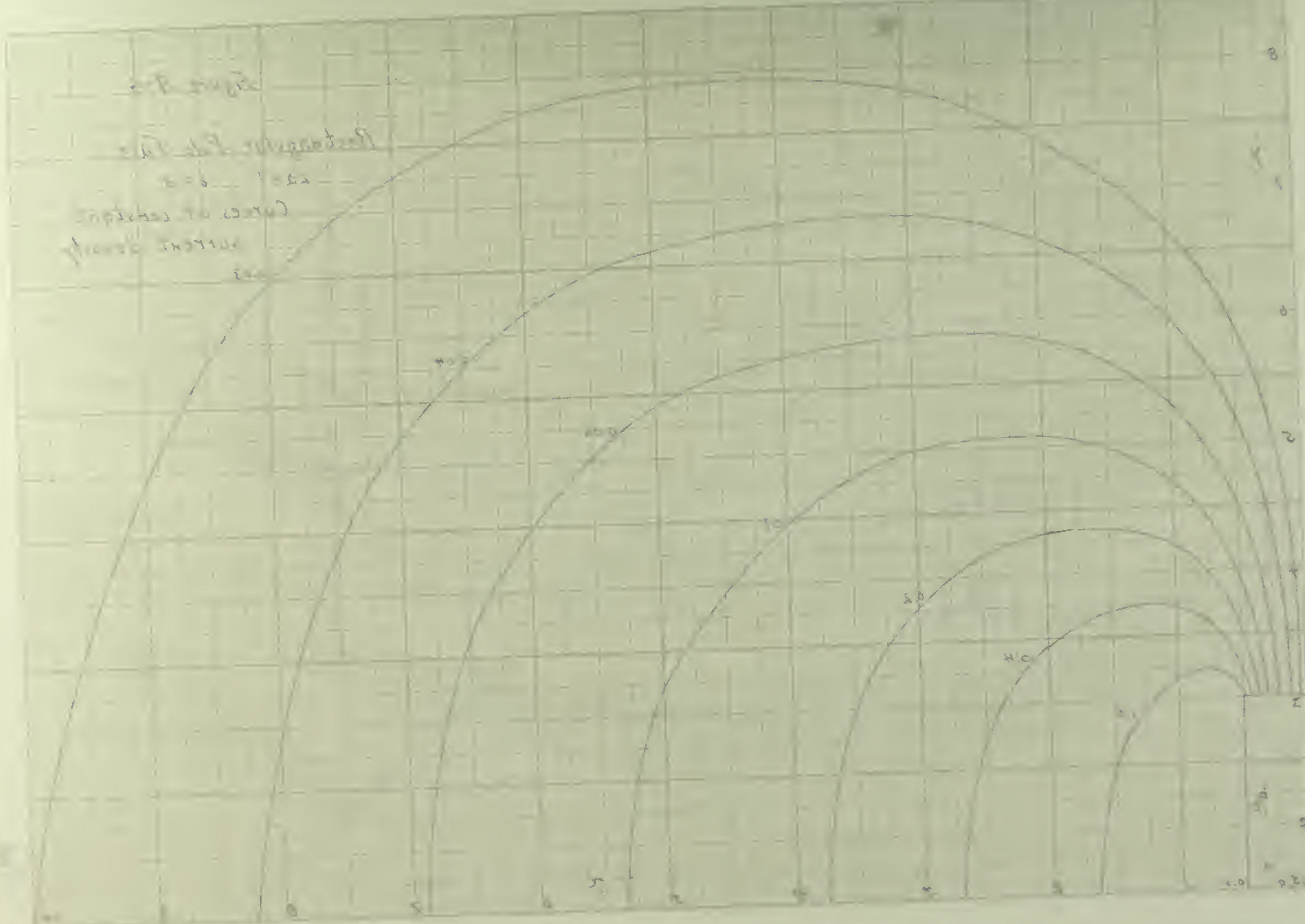


Figure A-6
Rectangular Pole Face
2a = 1 b = 3
Curves at constant current density
y
x
0.03
0.04
0.06
0.1
0.2
0.4
1.0
b/2
a
1.5, 0
0.5
1
2
3
4
5
6
7
8
9
10



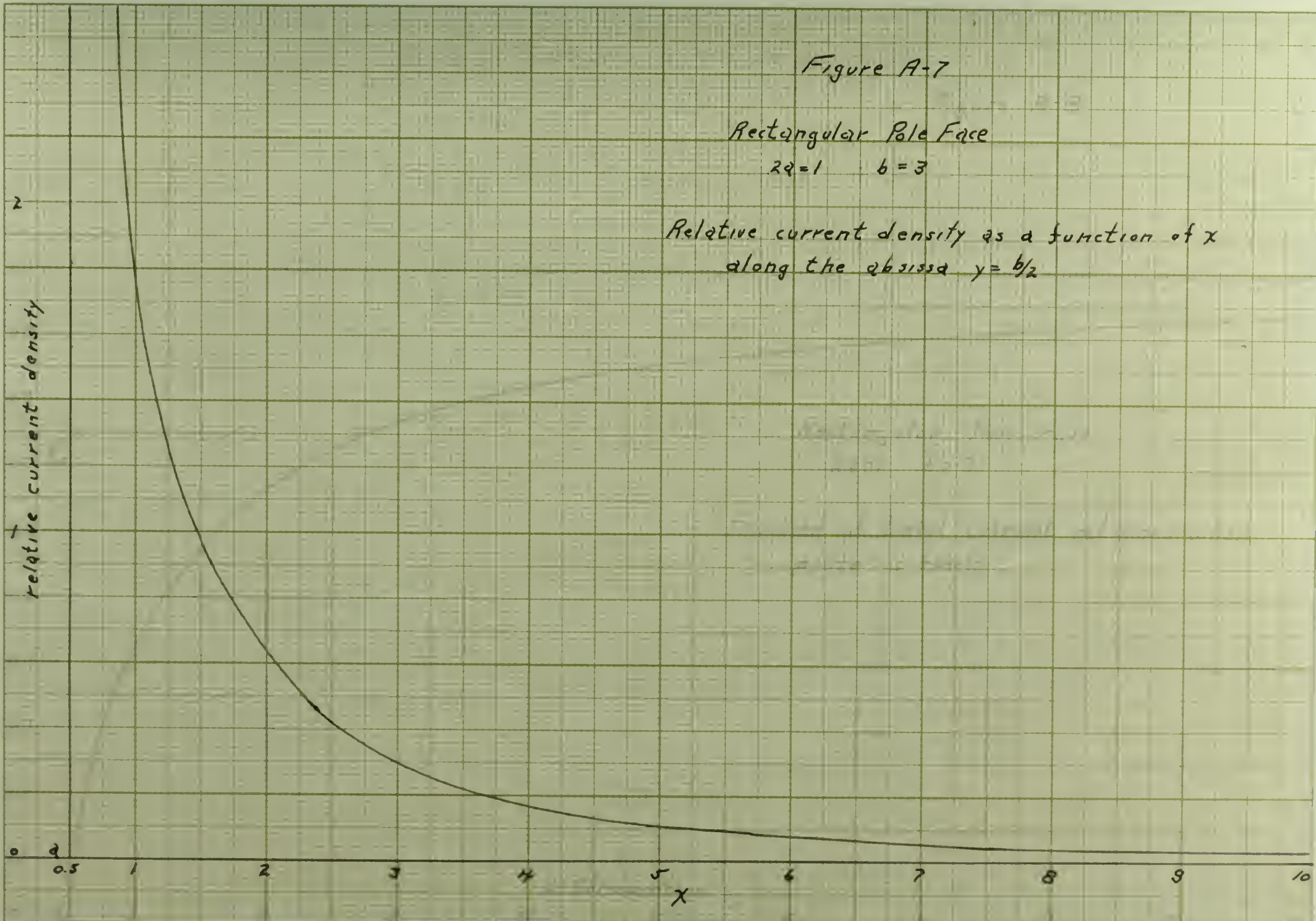


Figure A-7

Rectangular Pole Face

$2a = 1 \quad b = 3$

Relative current density as a function of x along the absissa $y = b/2$

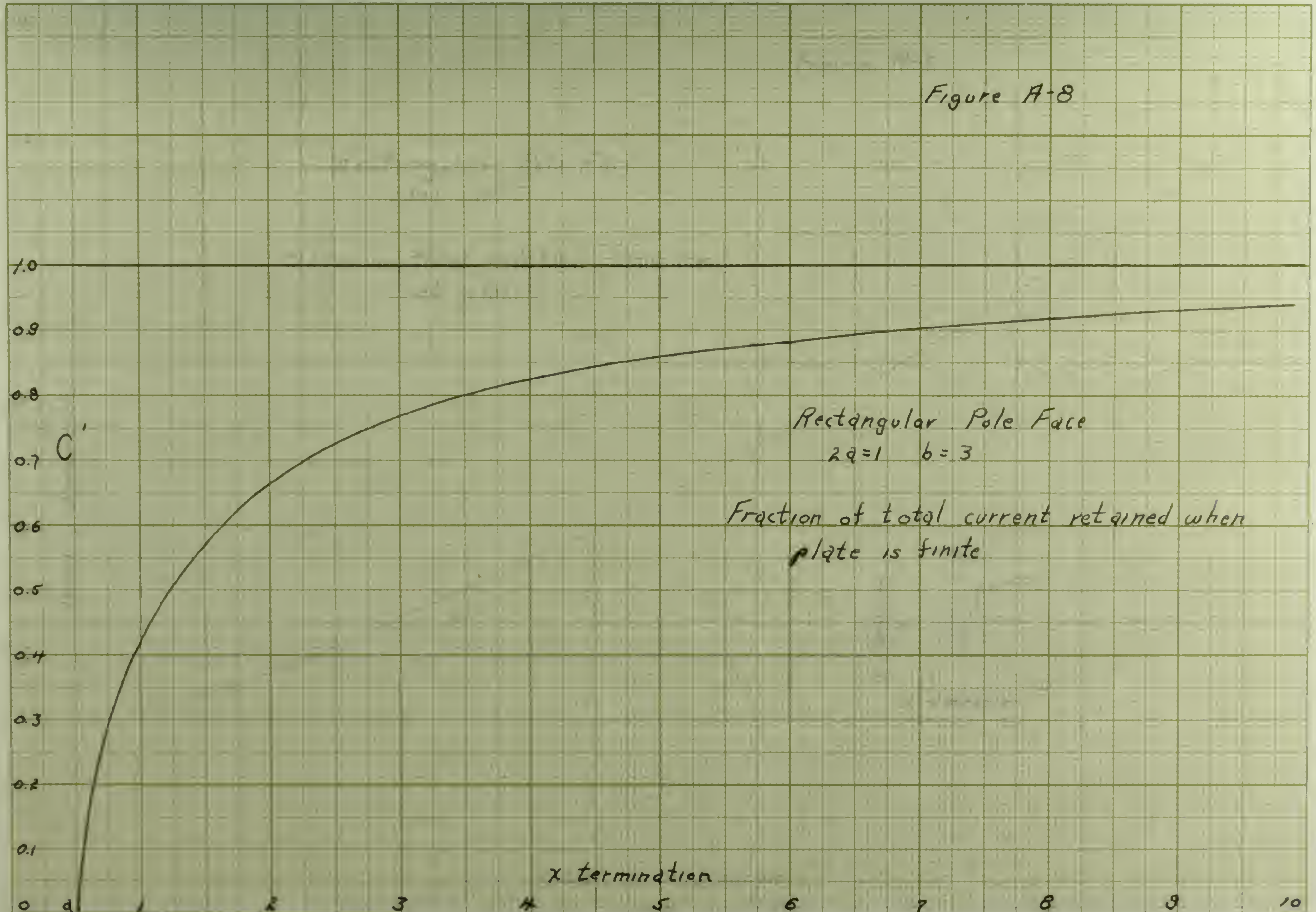


Figure A-8
Rectangular Pole Face
2a=1 b=3
Fraction of total current retained when plate is finite
C'
x termination
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
a
1
2
3
4
5
6
7
8
9
10

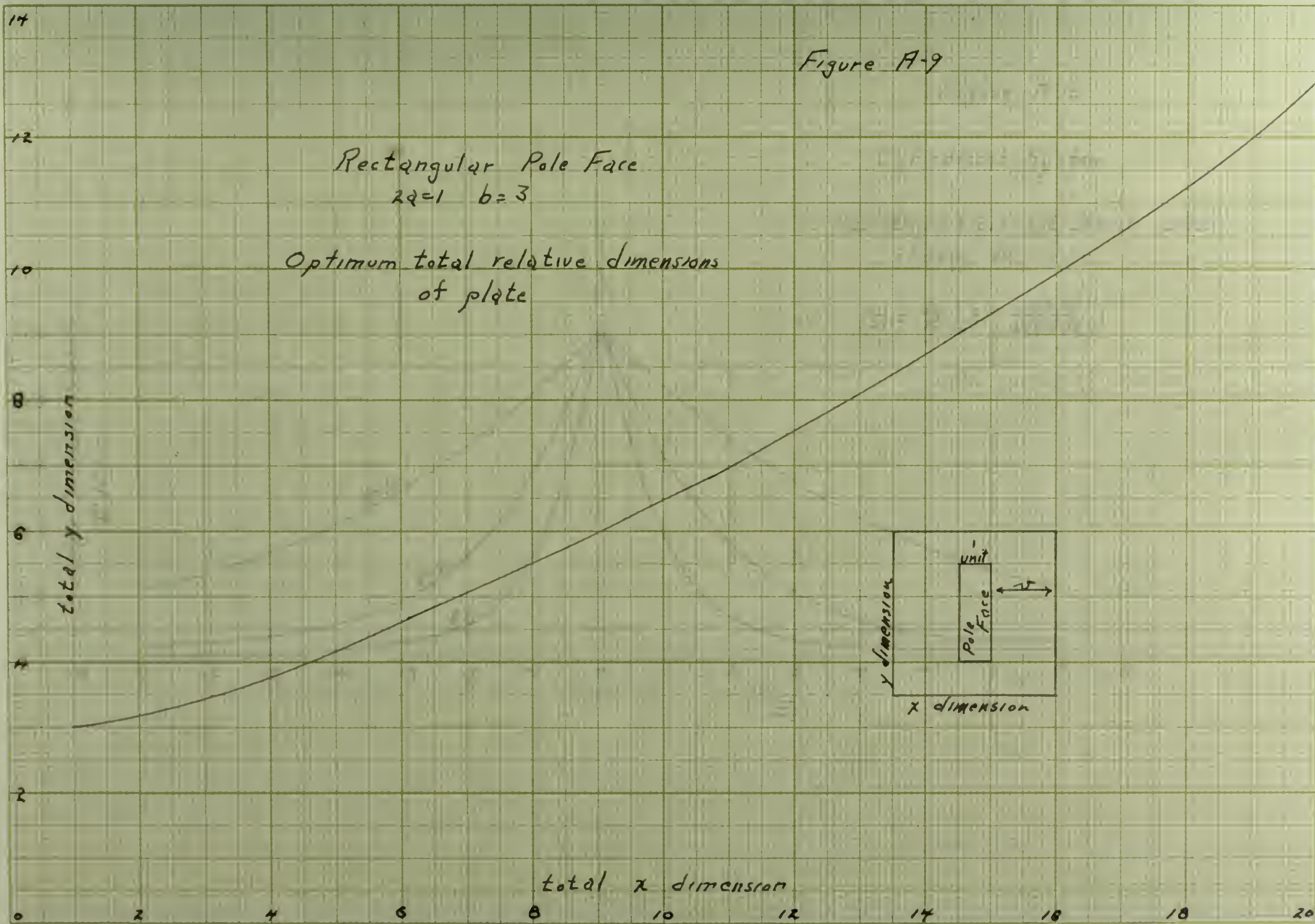

Rectangular Pole Face
$2a=1 \quad b=3$
Optimum total relative dimensions
of plate
total y dimension
total x dimension
unit
Pole Face
y dimension
x dimension
0
2
4
6
8
10
12
14
16
18
20
14
12
10
8
6
4
2

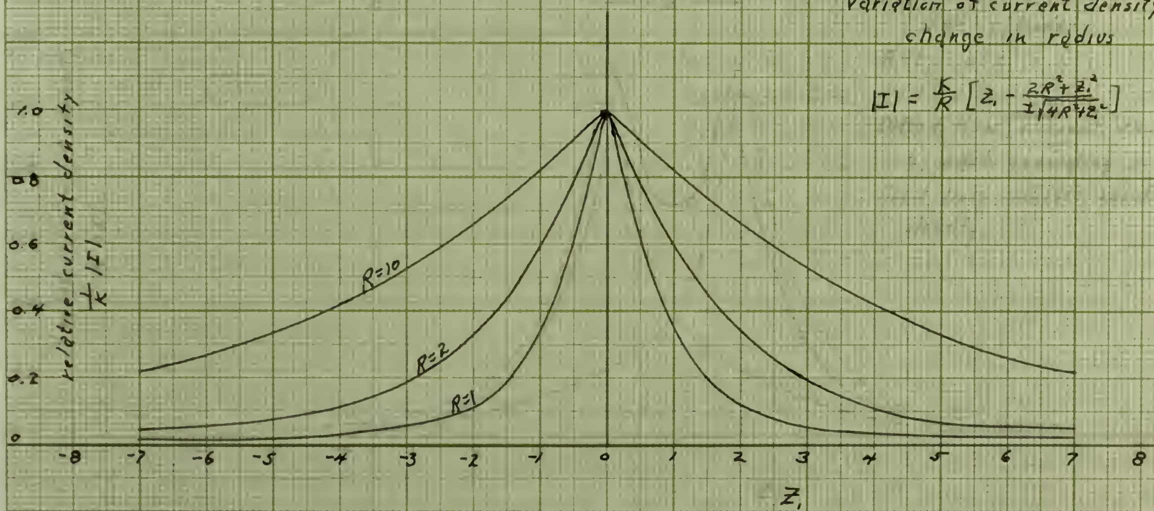
Figure A-9

Figure A-10

Cylindrical System

Variation of current density with change in radius

$$|I| = \frac{K}{R}\left[z_1 - \frac{2R^2+z_1^2}{2\sqrt{4R^2+z_1^2}}\right]$$



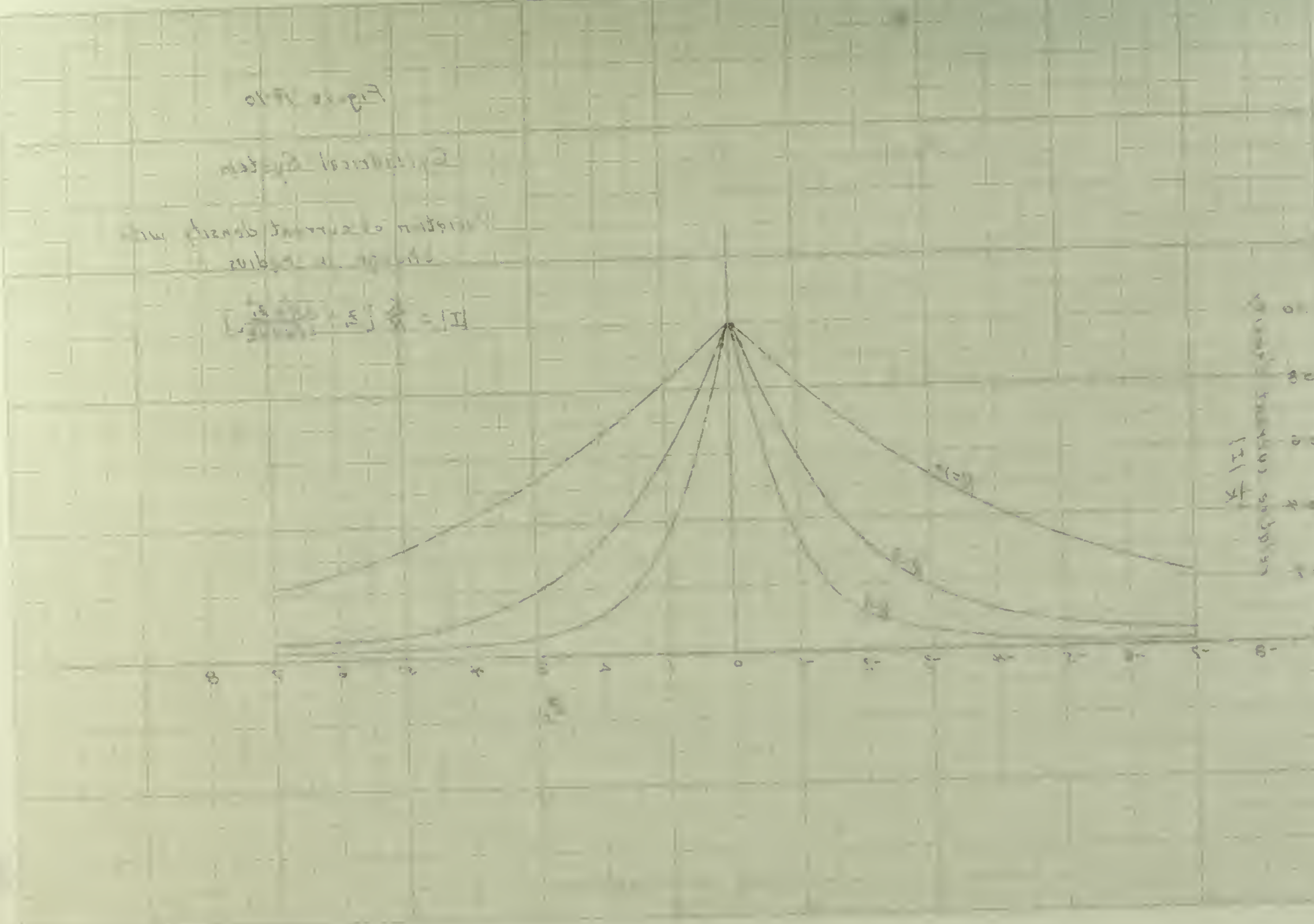


Figure 7-10
Cylindrical System
Variation of current density with change in radius

Figure A-11

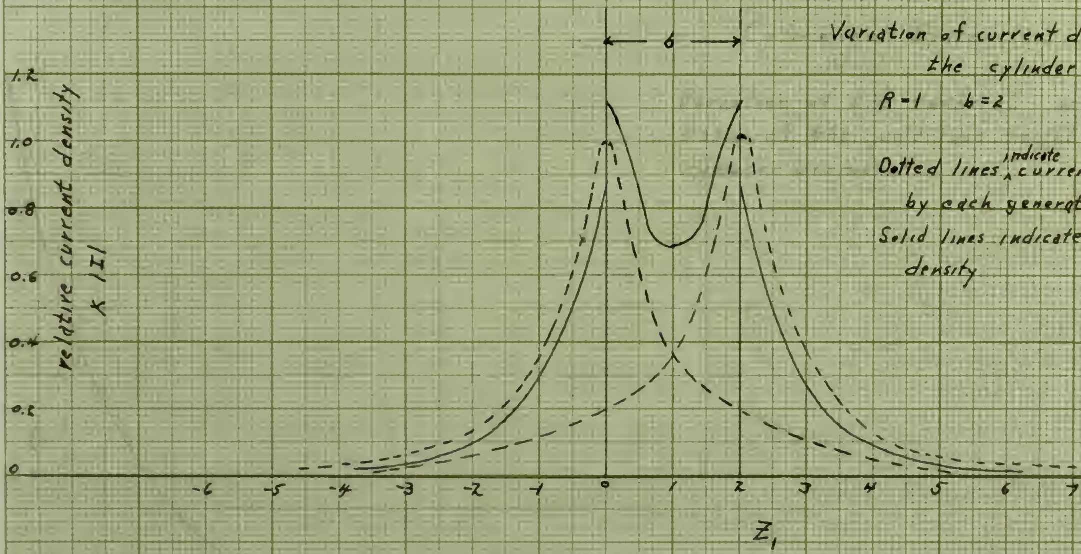
Cylindrical System

Variation of current density along the cylinder

R = 1   b = 2

Dotted lines indicate current density produced by each generating surface

Solid lines indicate resultant current density



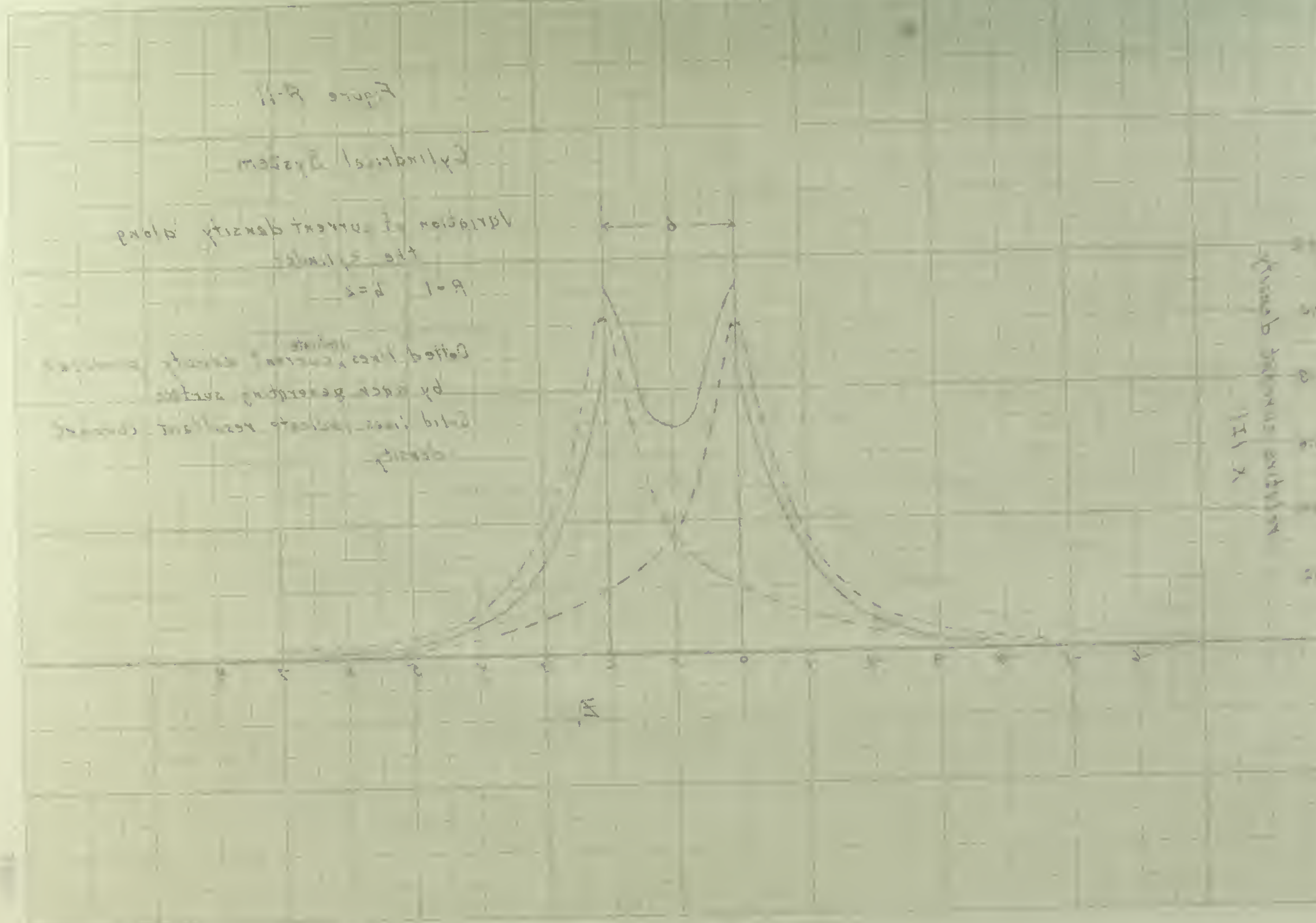


Figure R-11
Cylindrical System
Variation of current density along the cylinder
R=1 b=2
Dotted lines indicate current density produced by each generating surface
Solid lines indicate resultant current density
b
Z

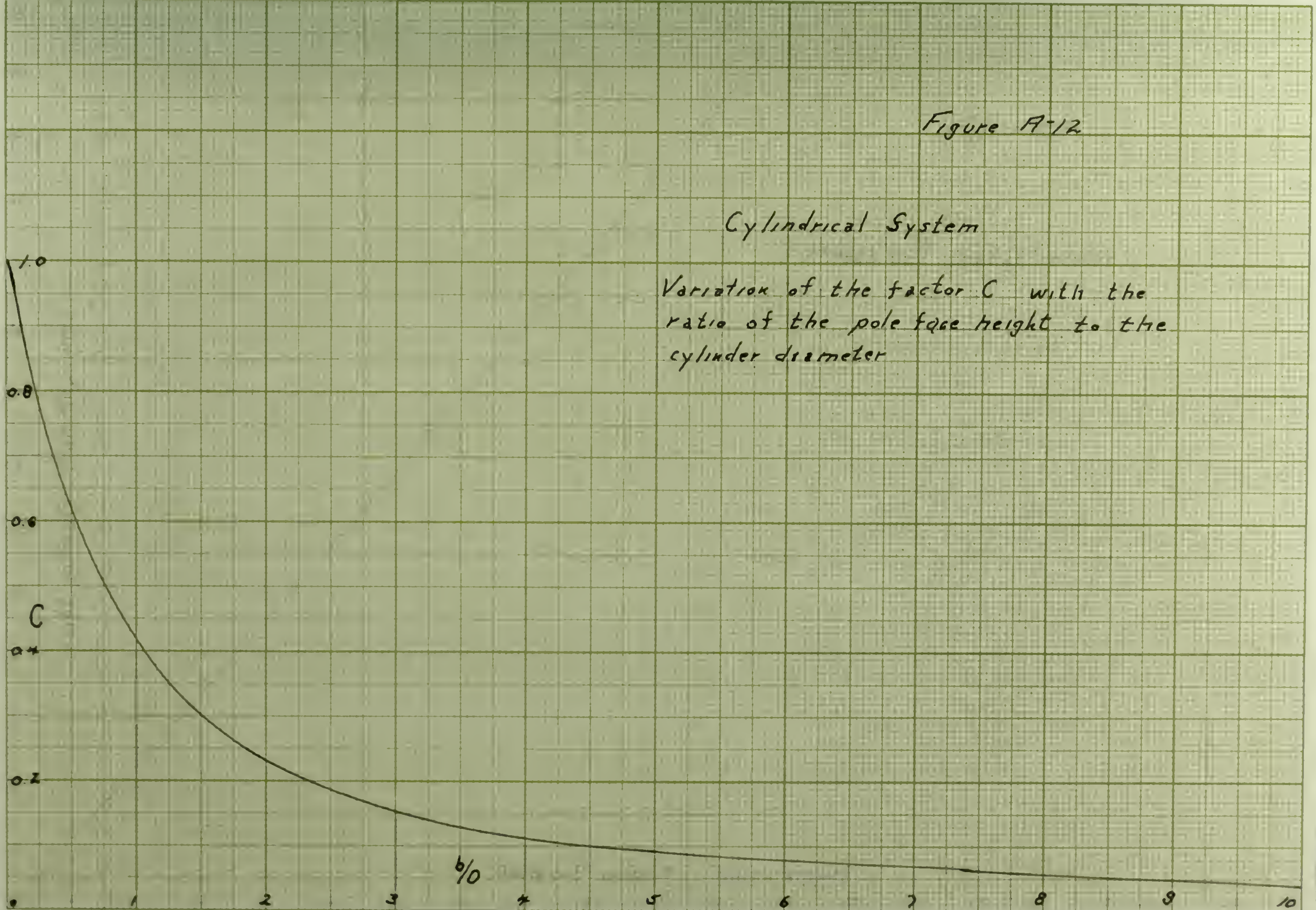
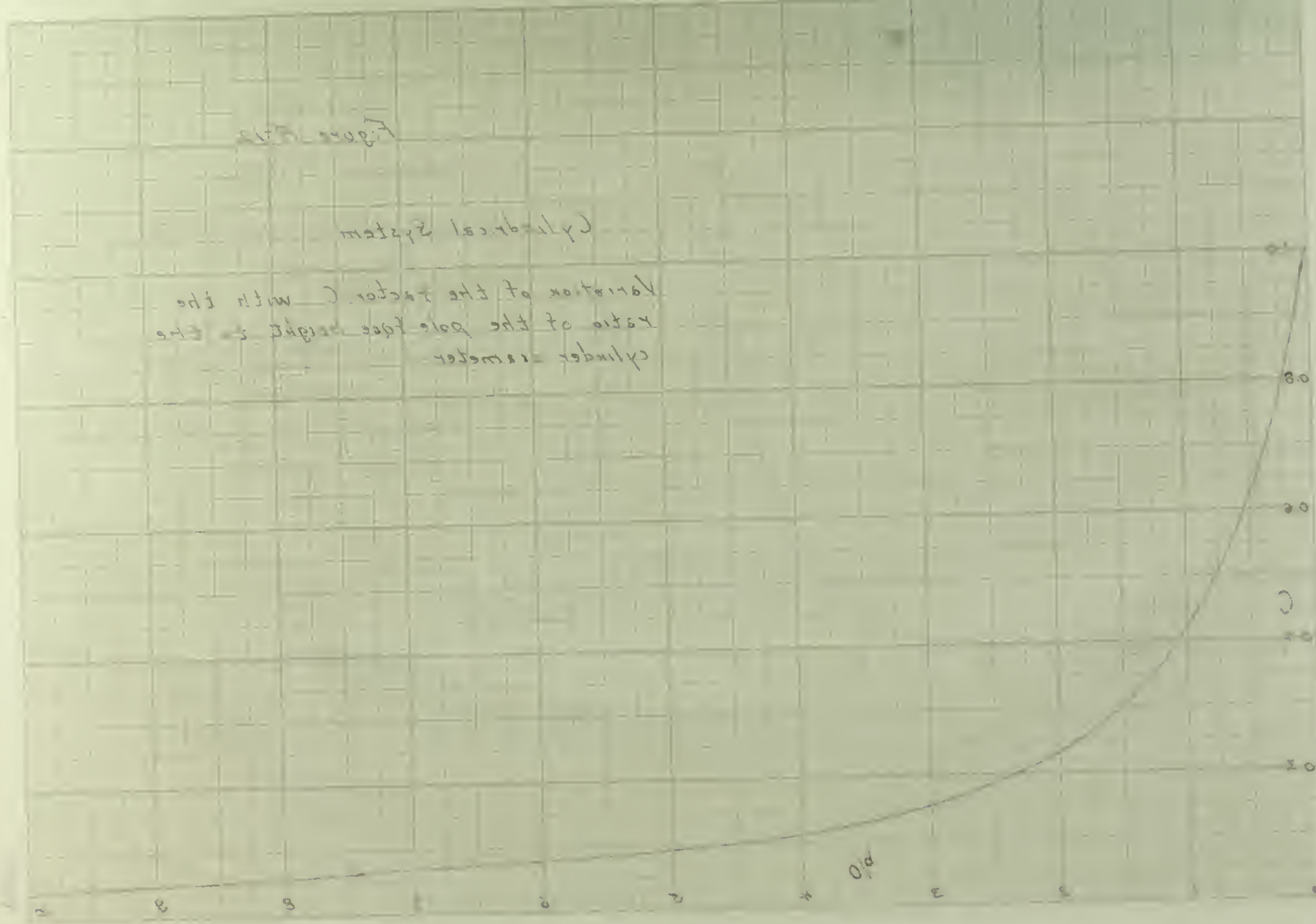


Figure A-12
Cylindrical System
Variation of the factor C with the ratio of the pole face height to the cylinder diameter
C
1.0
0.8
0.6
0.4
0.2
b/D
0
1
2
3
4
5
6
7
8
9
10


Cylindrical System
Variation of the factor C with the ratio of the pole face height to the cylinder diameter
C
p/D

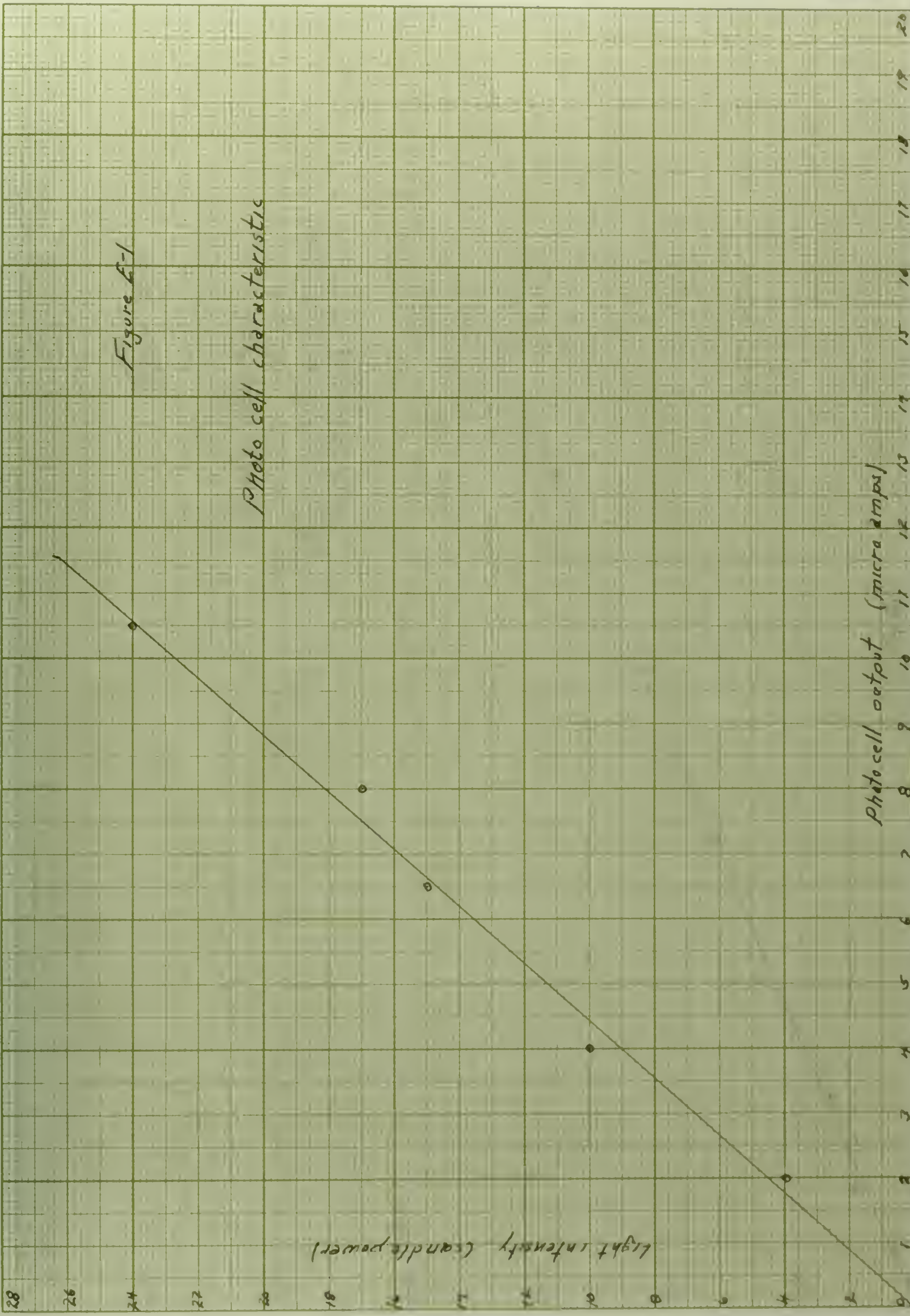
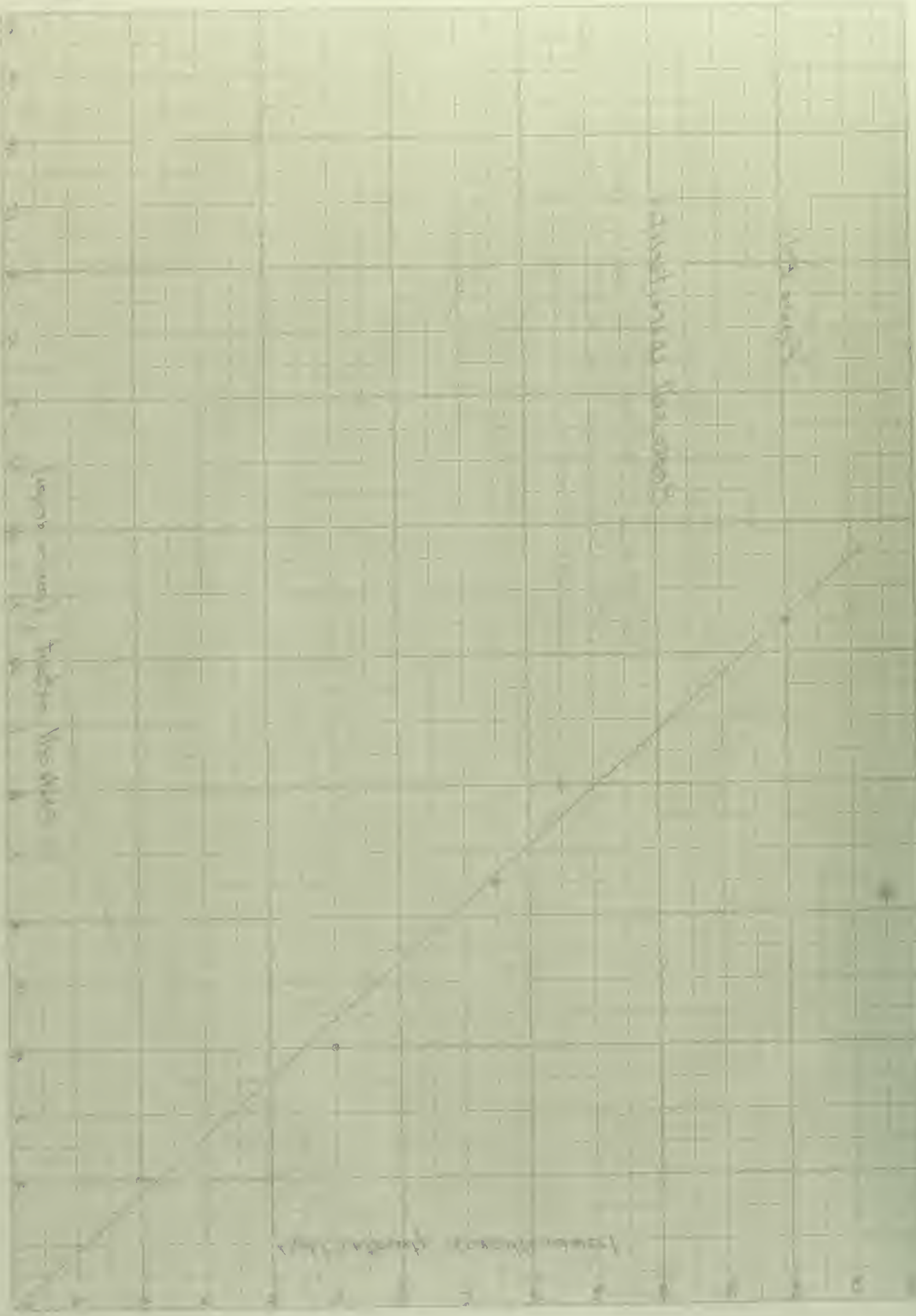


Figure E-1
Photo cell characteristic
Photo cell output (micro amps)
light intensity (candle power)
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
0 2 4 6 8 10 12 14 16 18 20 22 24 26 28



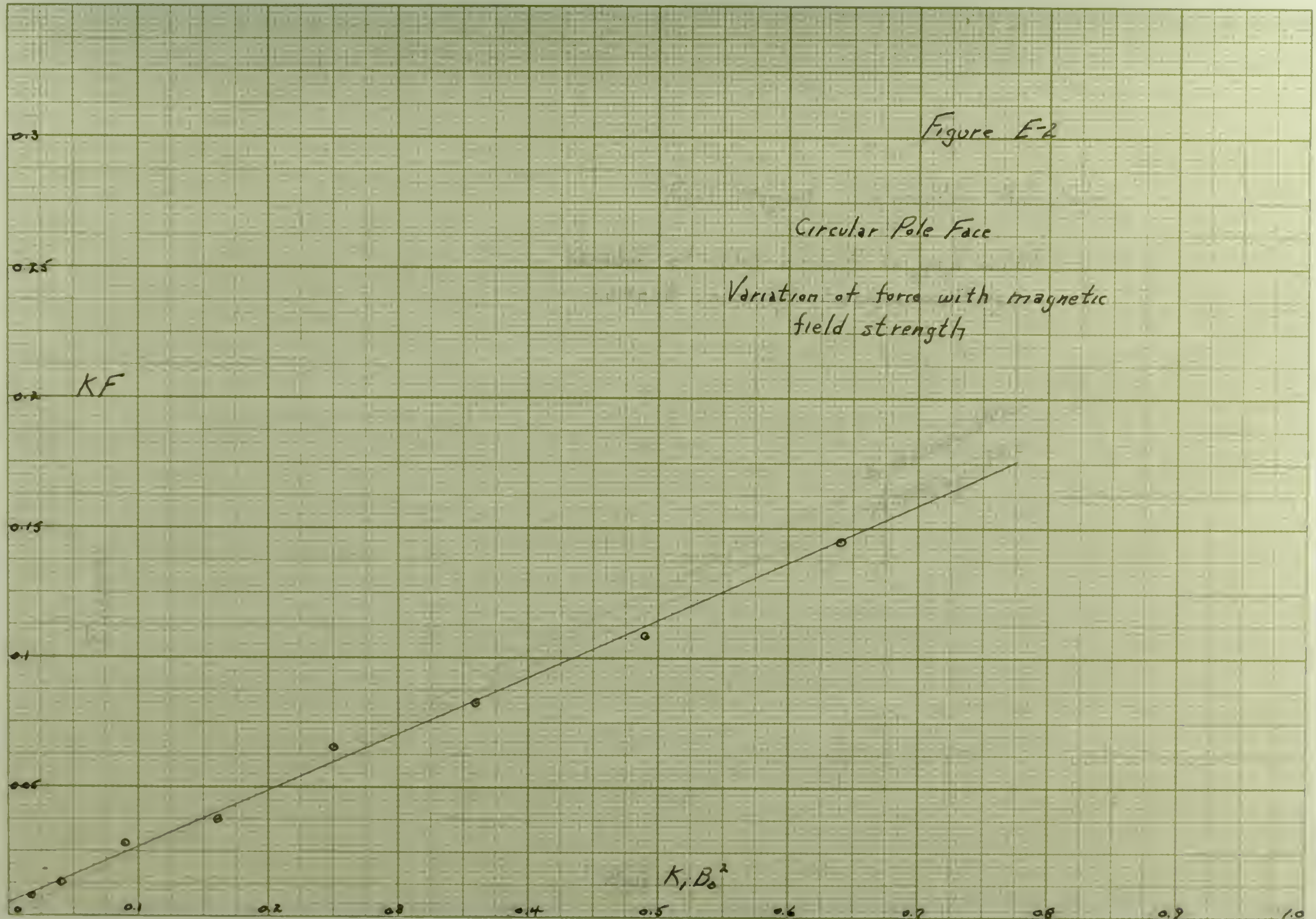


Figure E-2
Circular Pole Face
Variation of force with magnetic field strength
KF
$K_1 B_0^2$
0.3
0.25
0.2
0.15
0.1
0.05
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0

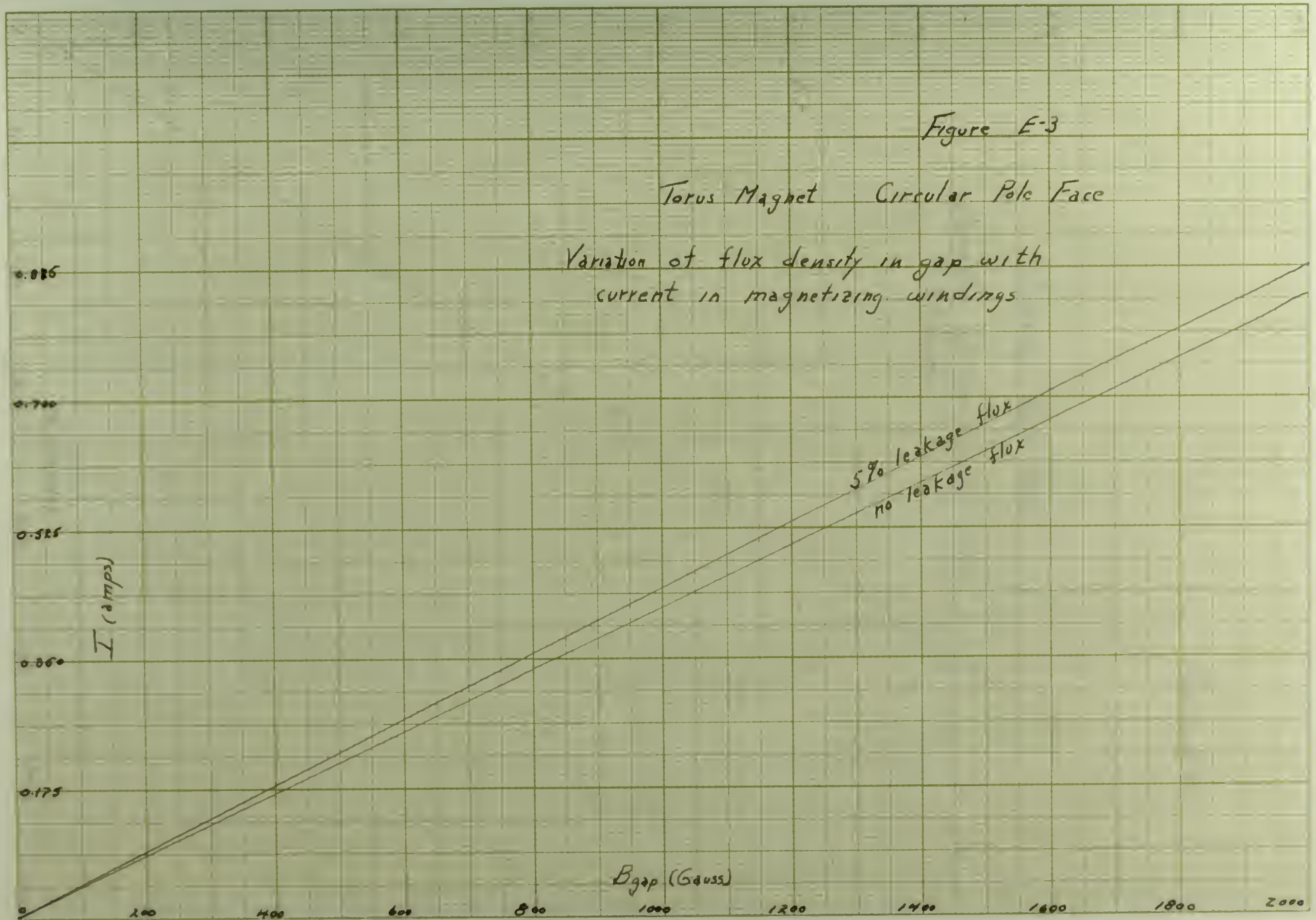
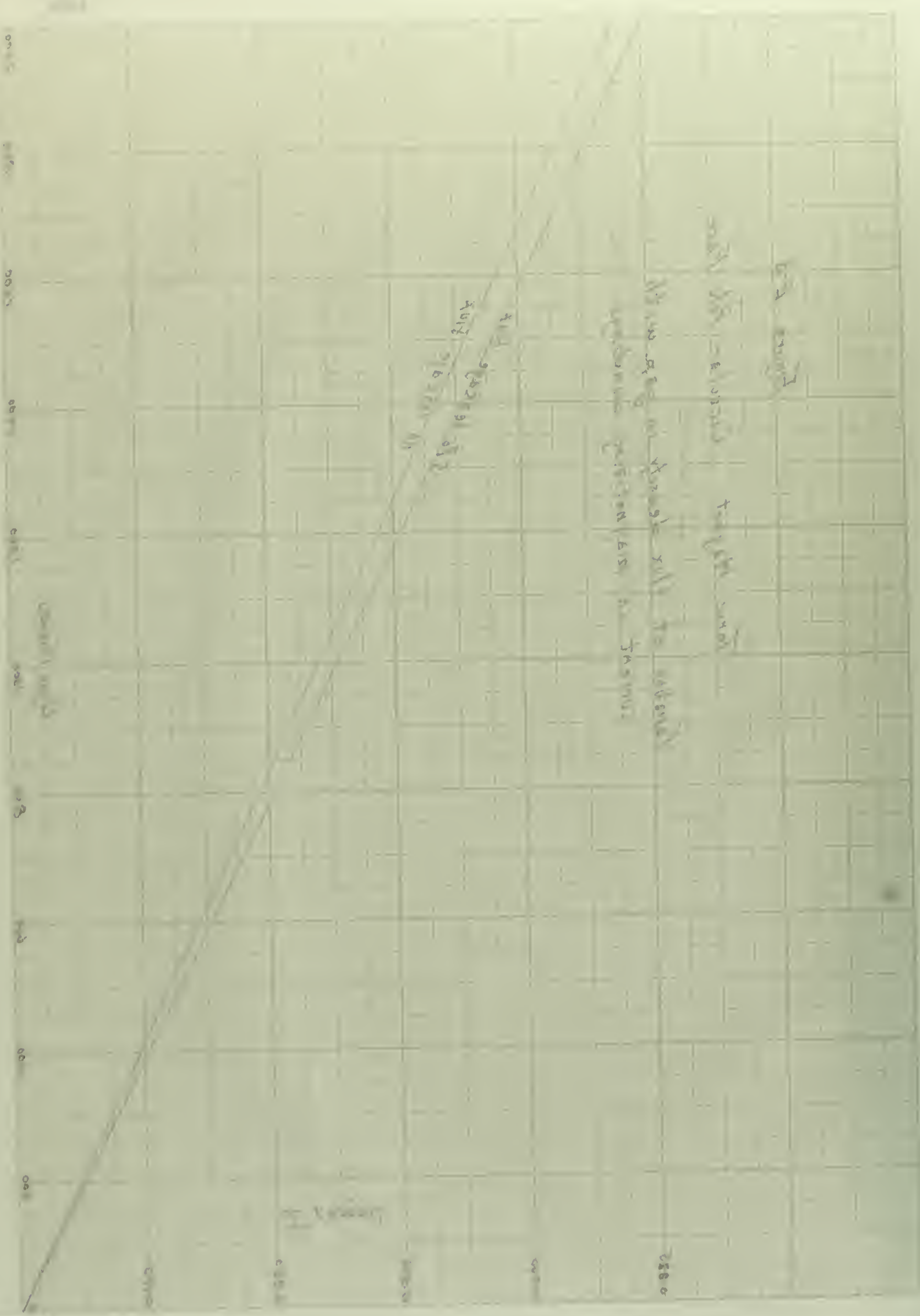


Figure E-3
Torus Magnet Circular Pole Face
Variation of flux density in gap with current in magnetizing windings
5% leakage flux
no leakage flux
I (amps)
0.875
0.700
0.525
0.350
0.175
Bgap (Gauss)
0
200
400
600
800
1000
1200
1400
1600
1800
2000



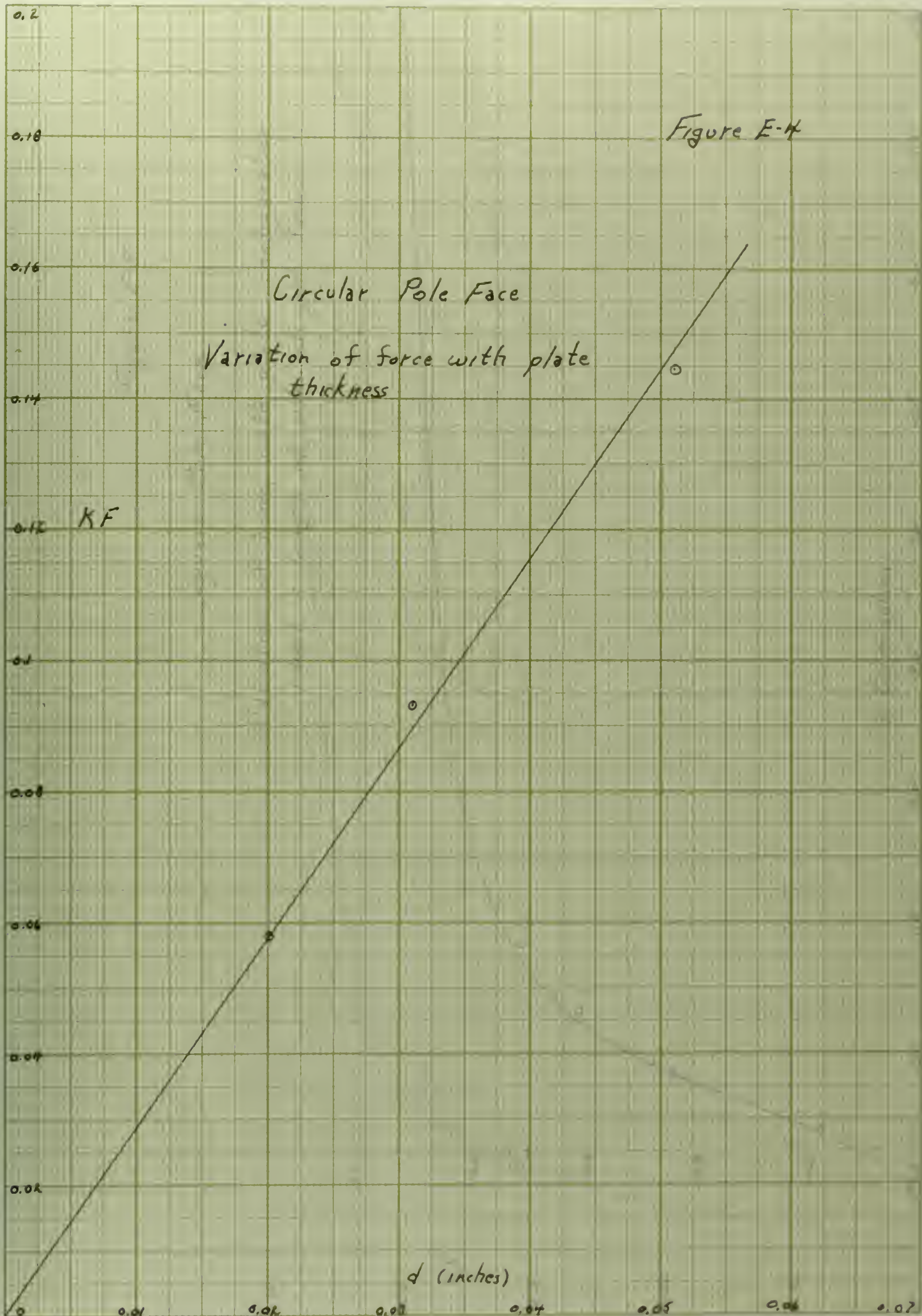
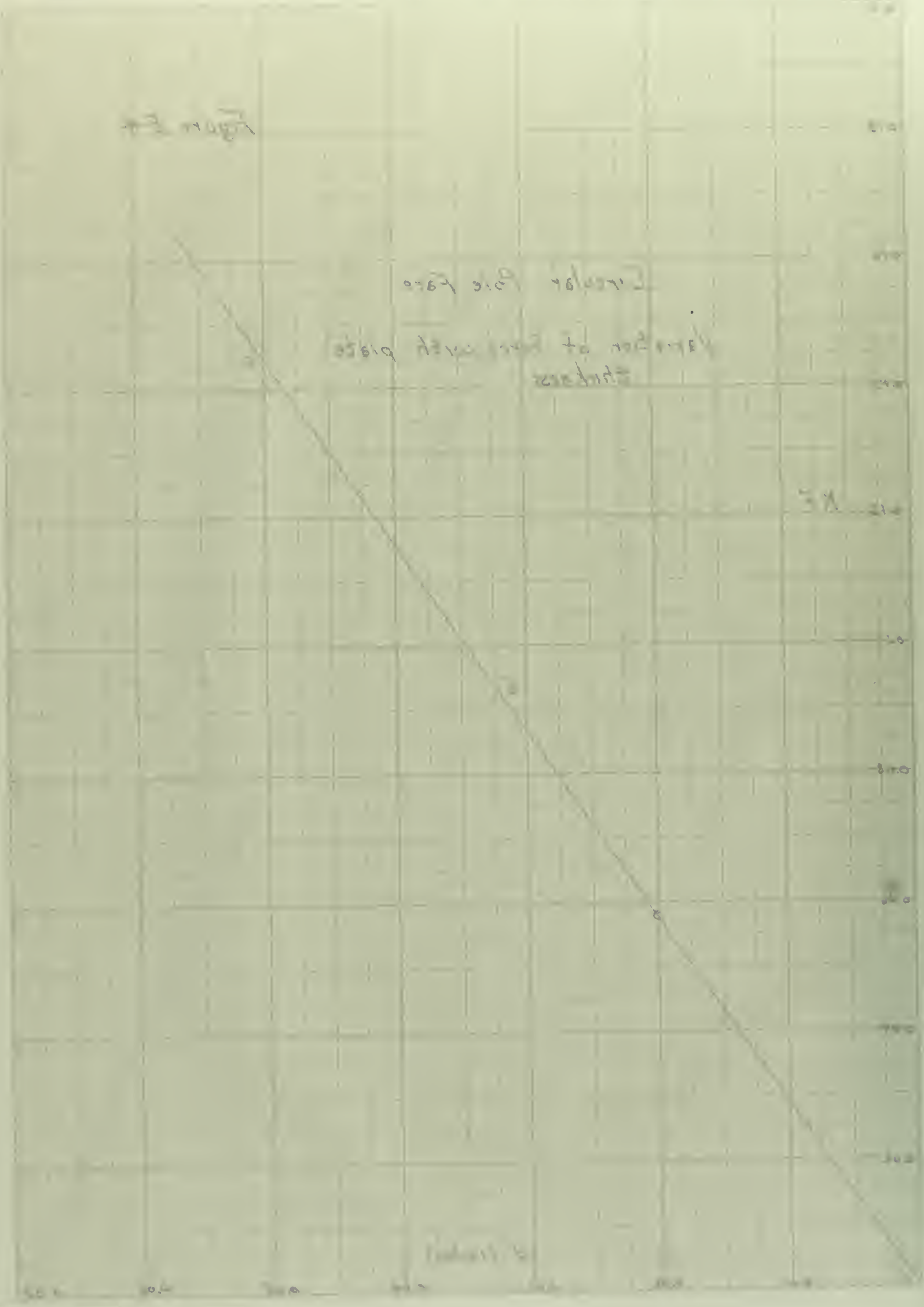


Figure E-4
Circular Pole Face
Variation of force with plate thickness
KF
d (inches)
0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07



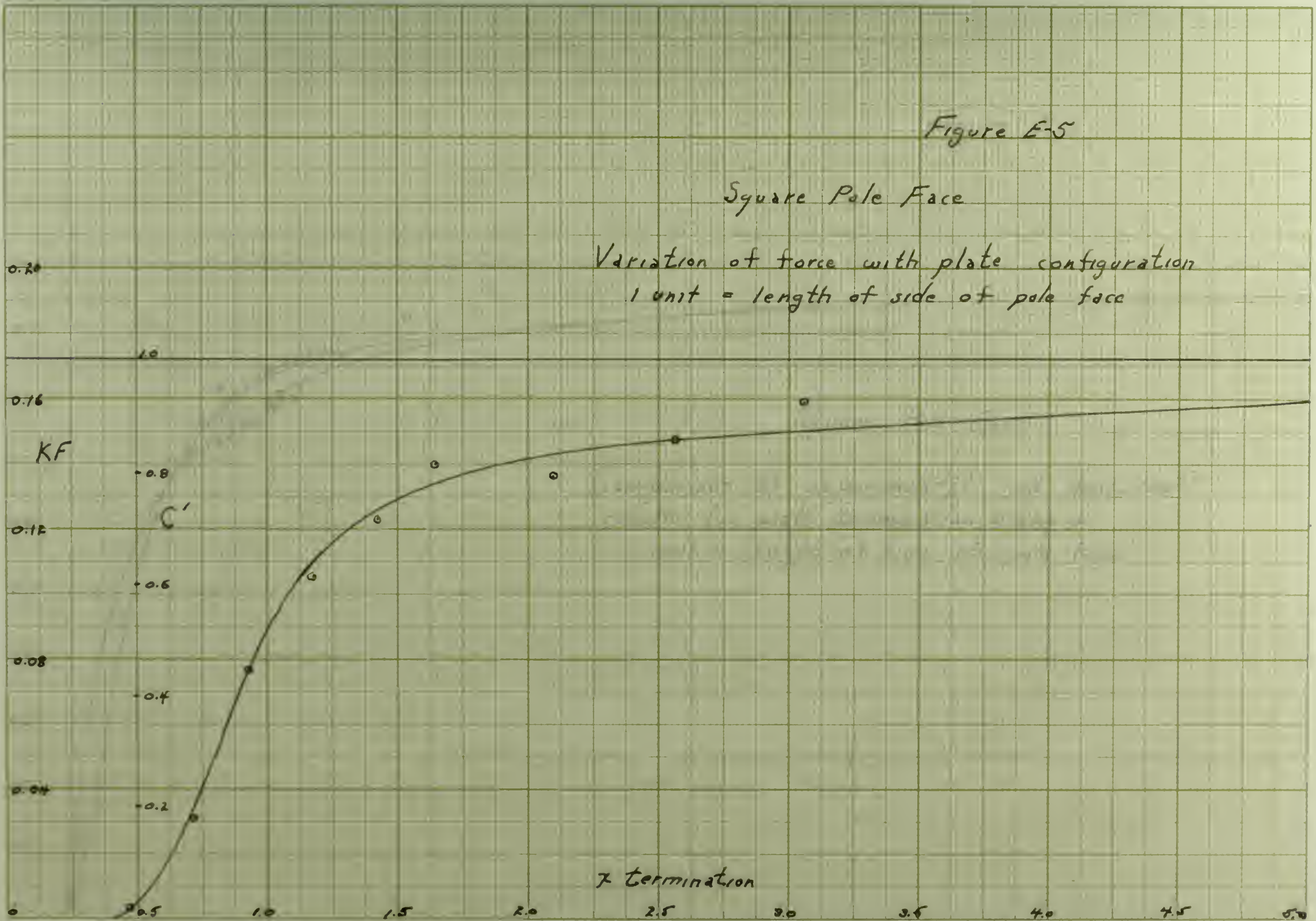
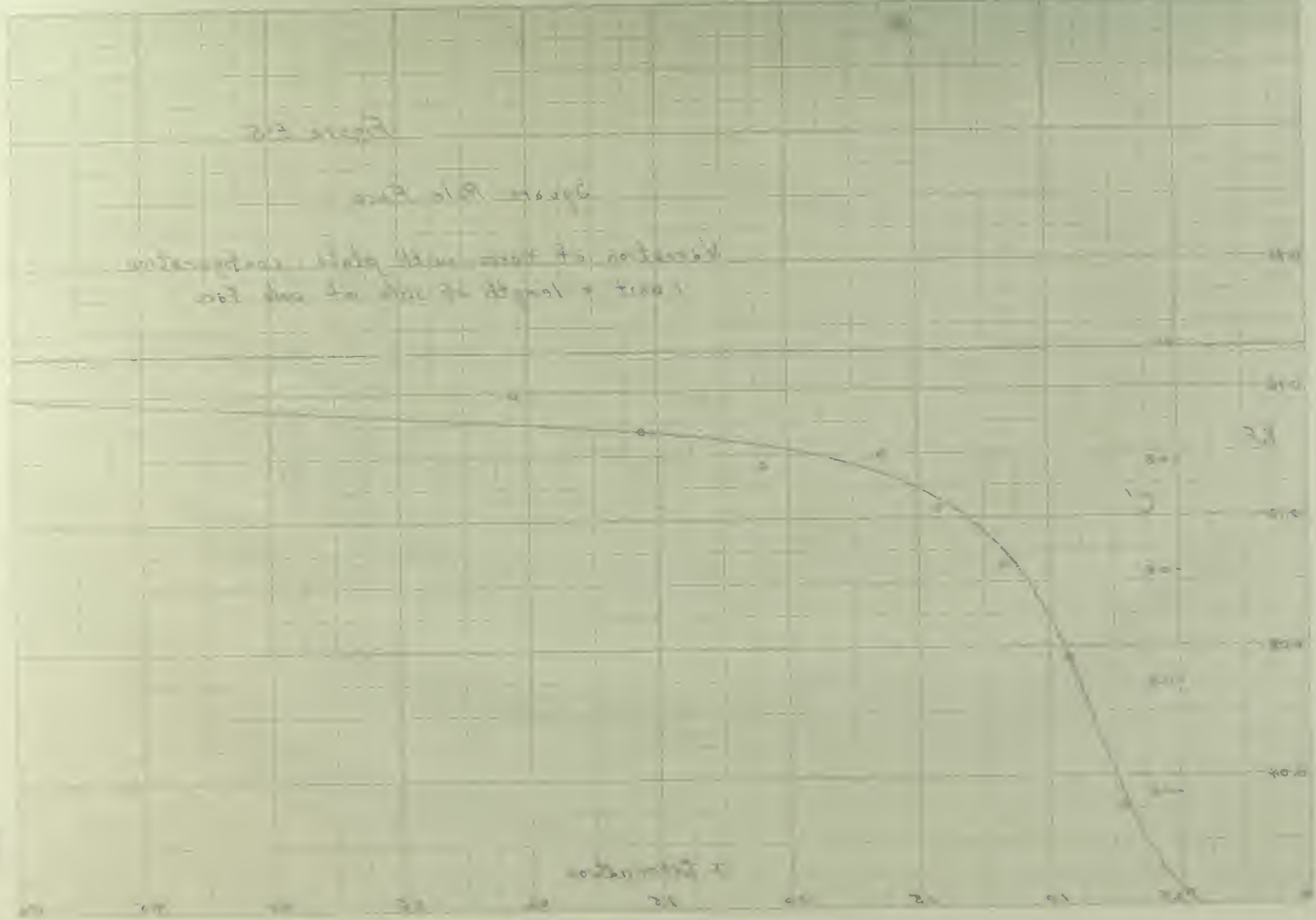


Figure E-5
Square Pole Face
Variation of force with plate configuration
1 unit = length of side of pole face
KF
C'
0.20
0.16
0.12
0.08
0.04
1.0
0.8
0.6
0.4
0.2
0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
x termination



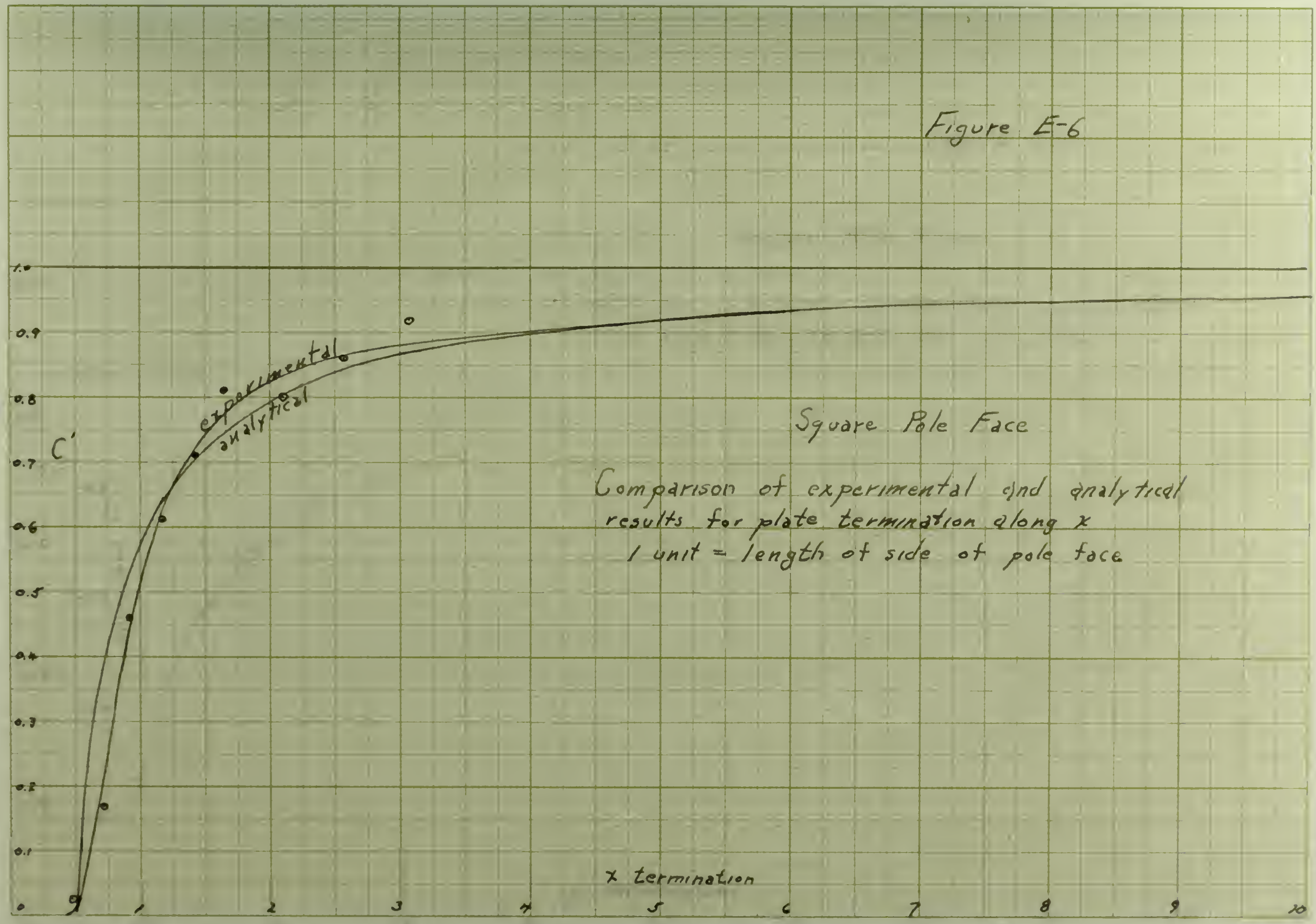


Figure E-6
Square Pole Face
Comparison of experimental and analytical
results for plate termination along x
1 unit = length of side of pole face
experimental
analytical
C'
x termination
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
1
2
3
4
5
6
7
8
9
10

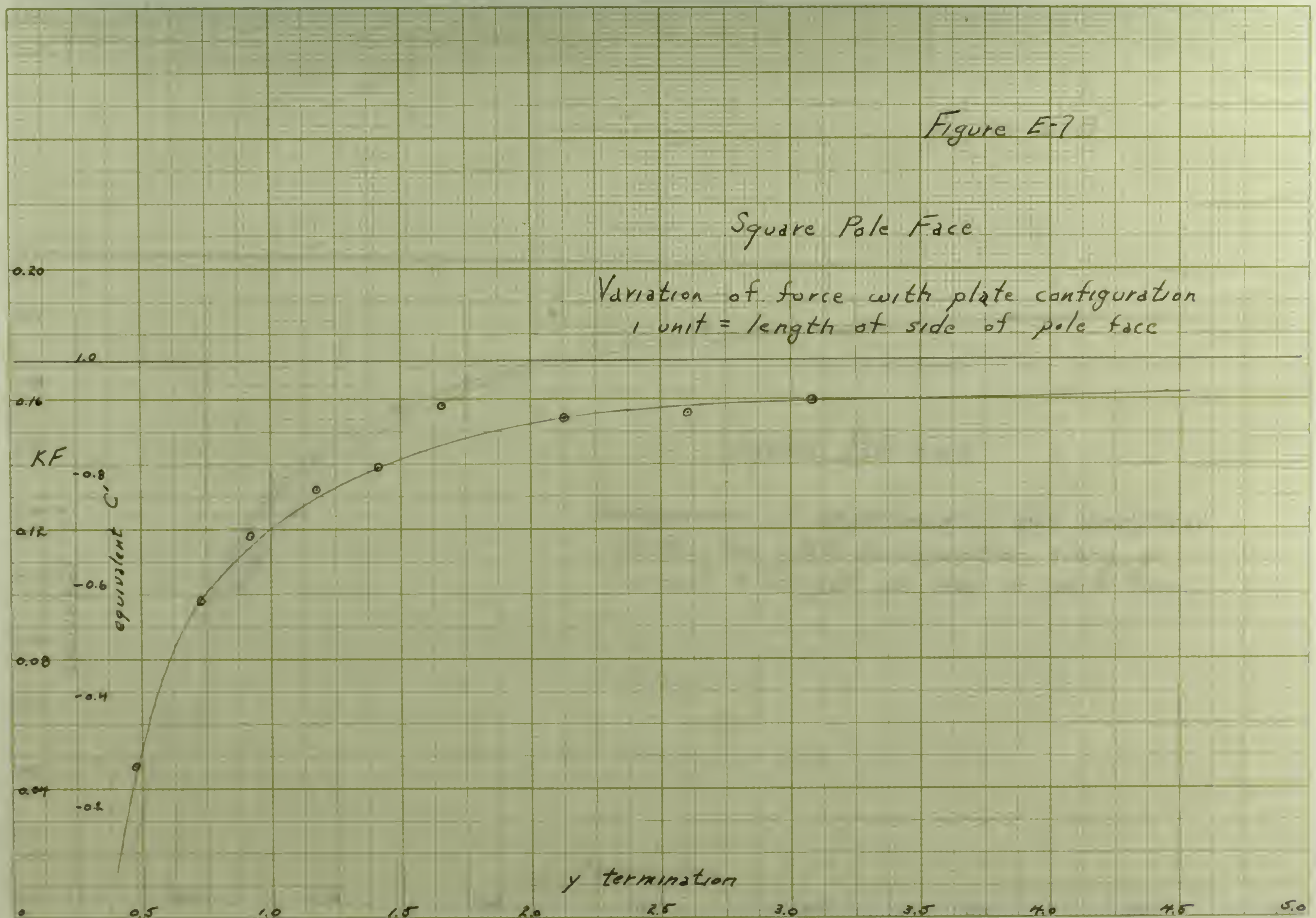


Figure E-7
Square Pole Face
Variation of force with plate configuration
1 unit = length of side of pole face
KF
equivalent C'
y termination
0.20
1.0
0.16
-0.8
0.12
-0.6
0.08
-0.4
0.04
-0.2
0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0

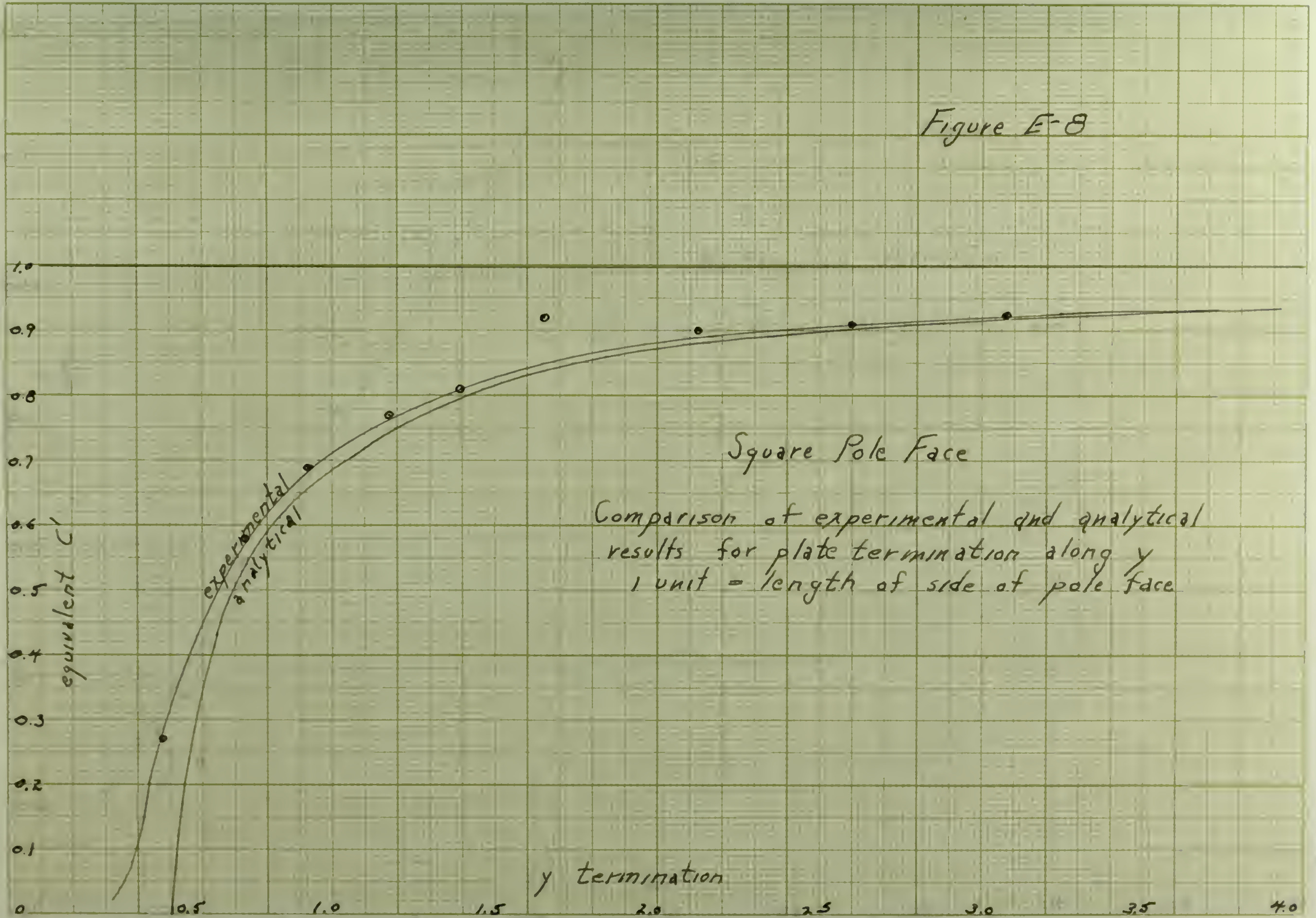
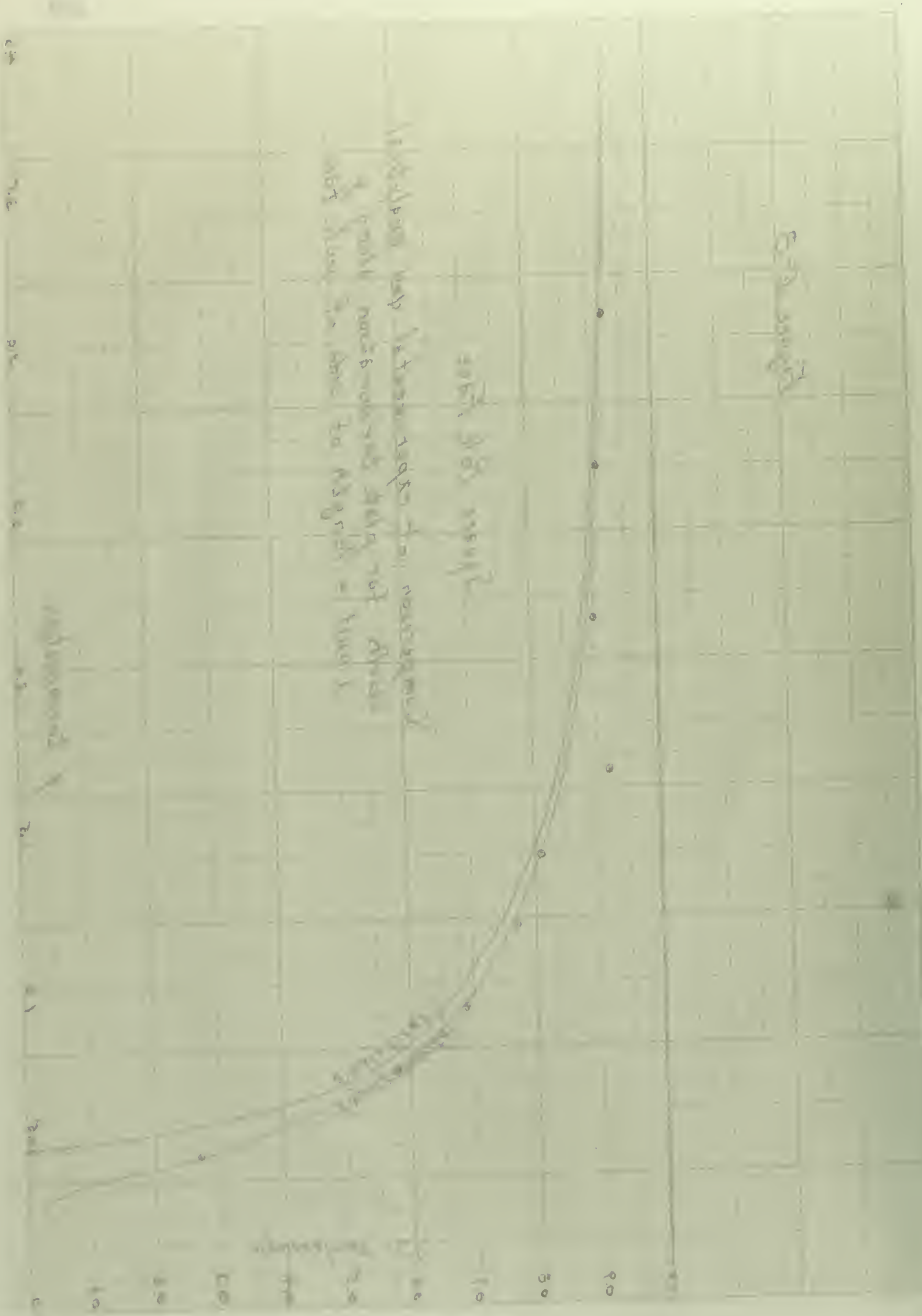


Figure E-8
Square Pole Face
Comparison of experimental and analytical
results for plate termination along y
1 unit = length of side of pole face
experimental
analytical
equivalent C'
y termination
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0



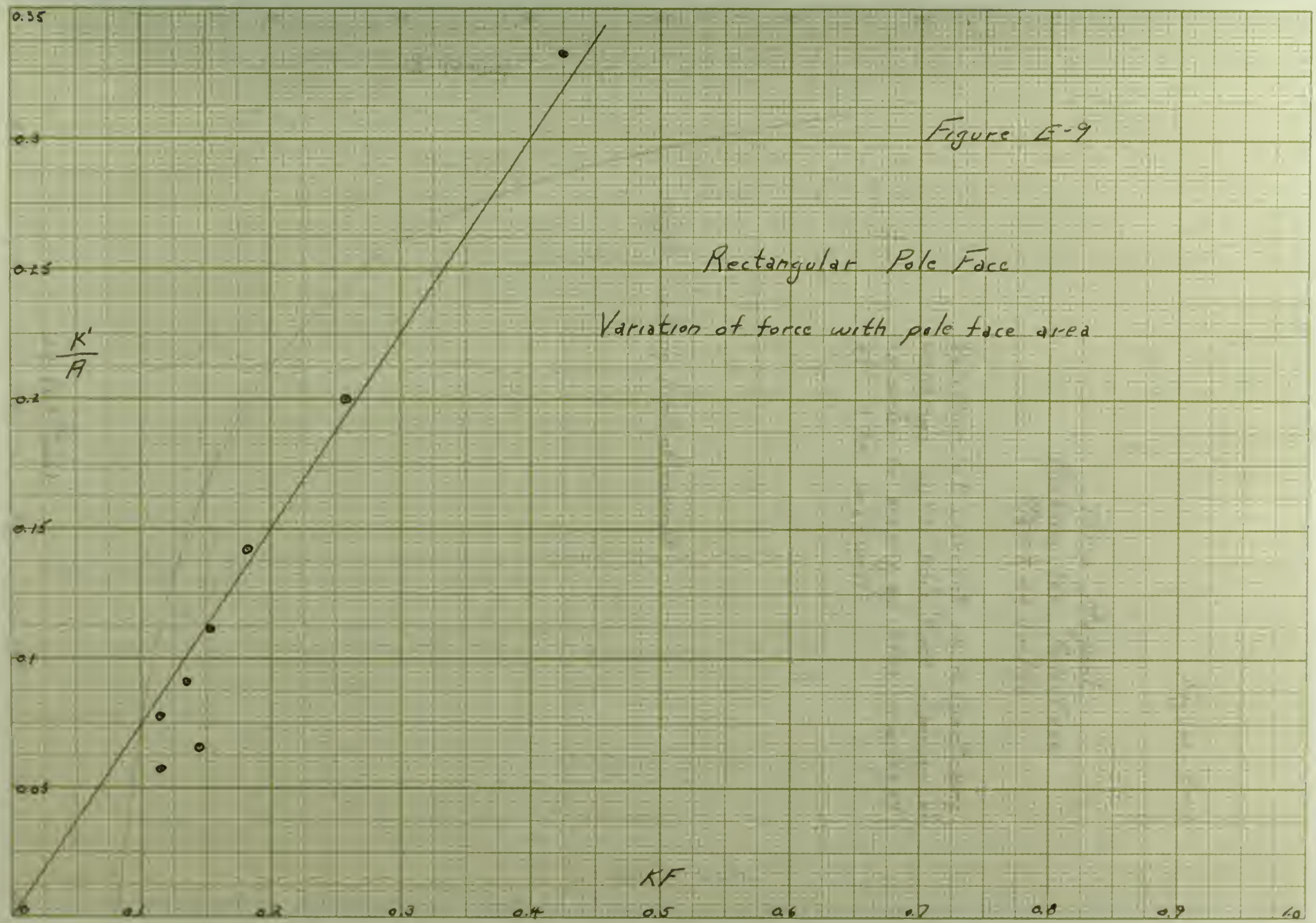


Figure E-9
Rectangular Pole Face
Variation of force with pole face area
K'/A
KF
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0

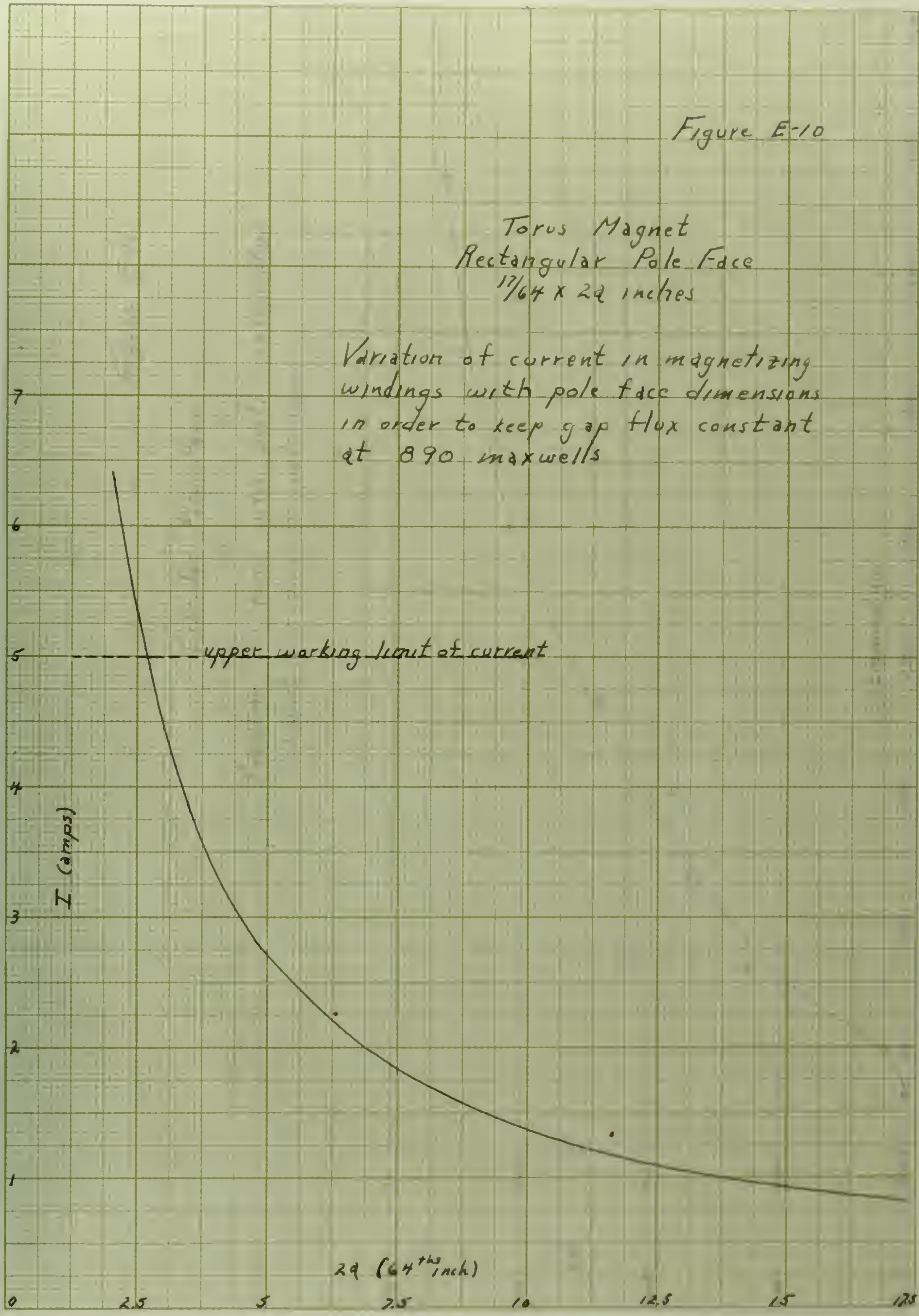
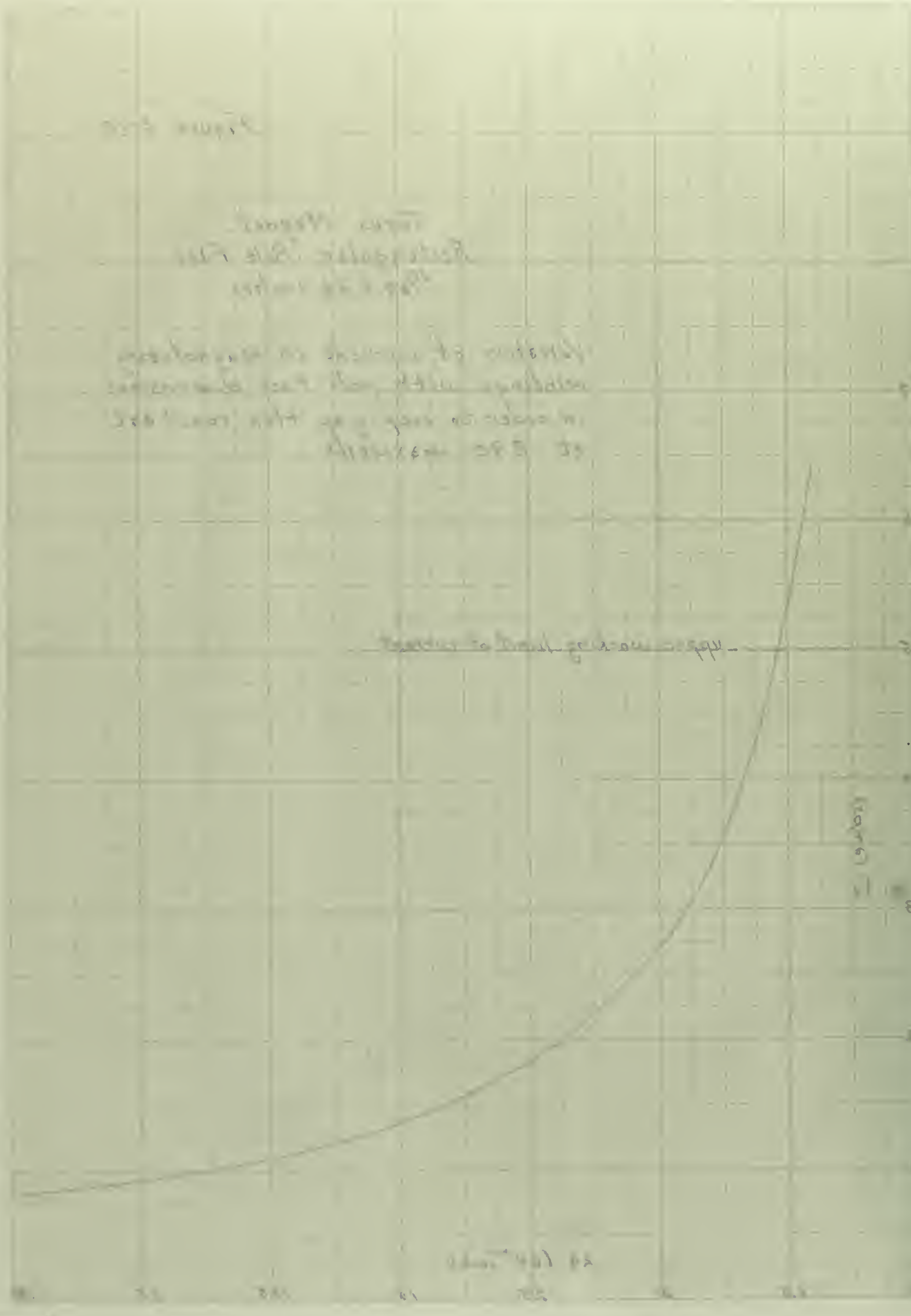


Figure E-10
Torus Magnet
Rectangular Pole Face
17/64 x 2a inches
Variation of current in magnetizing windings with pole face dimensions in order to keep gap flux constant at 890 maxwells
upper working limit of current
I (amps)
2a (64ths inch)
0
2.5
5
7.5
10
12.5
15
17.5
1
2
3
4
5
6
7



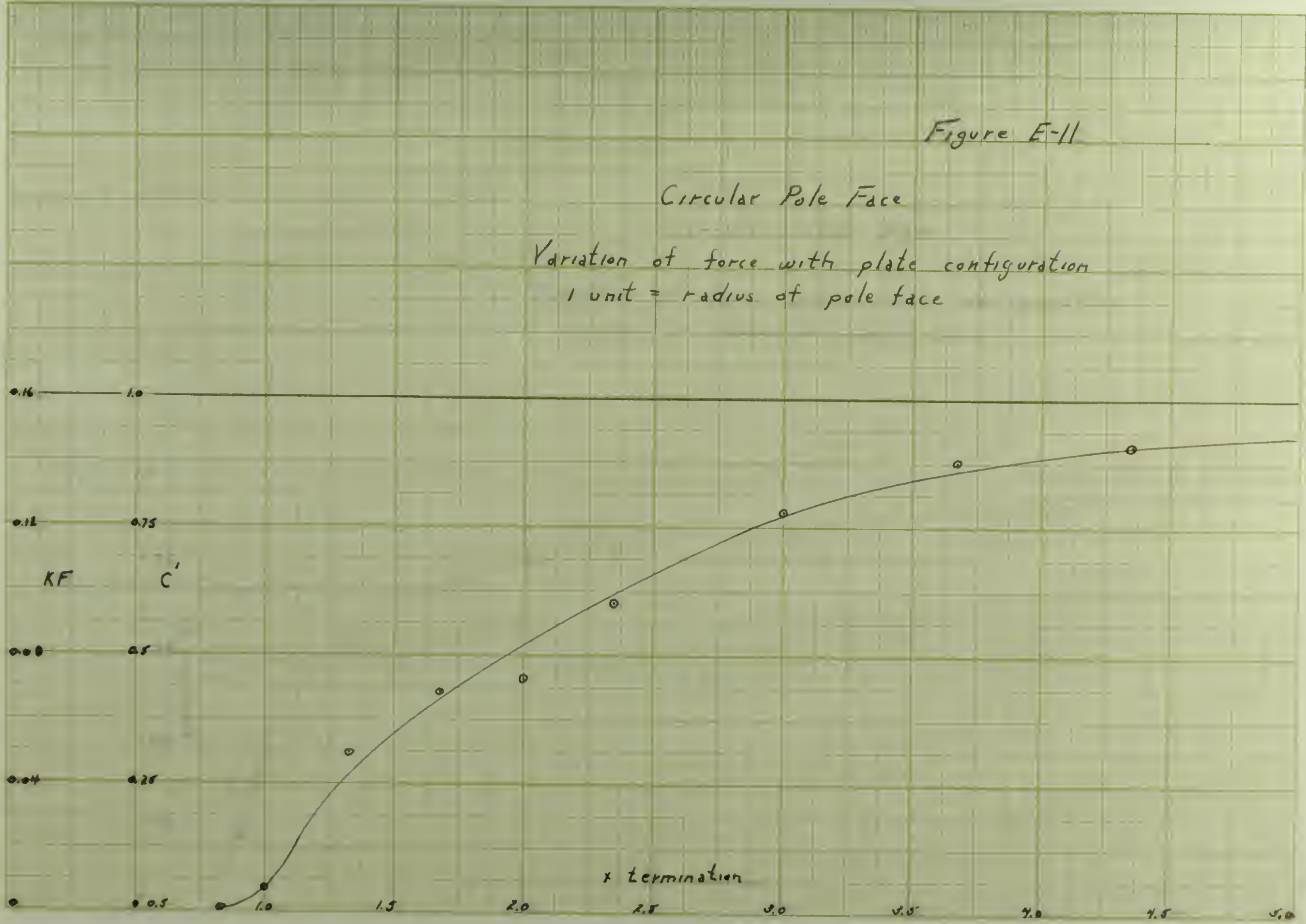


Figure E-11

Circular Pole Face

Variation of force with plate configuration
1 unit = radius of pole face

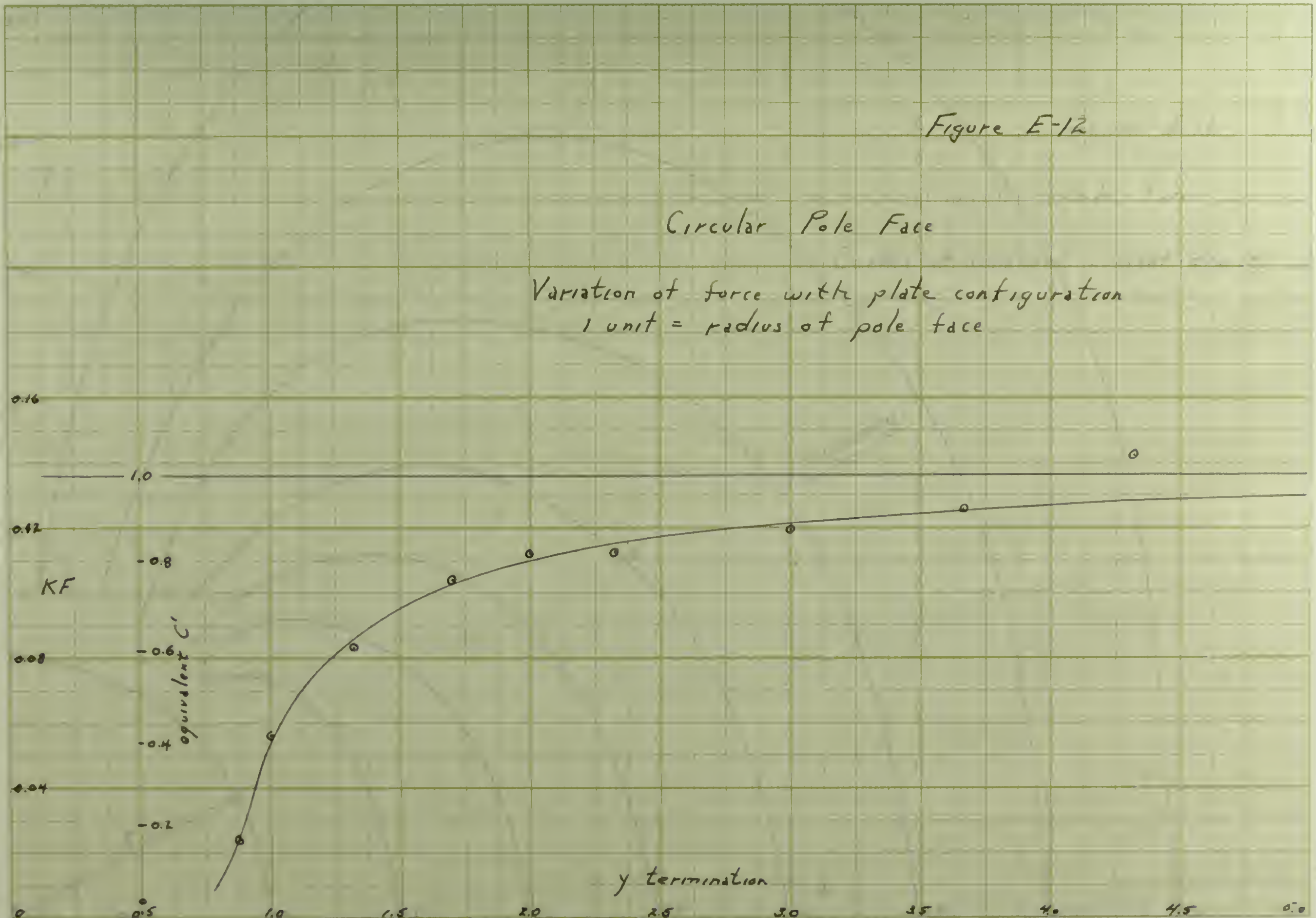


Figure E-12
Circular Pole Face
Variation of force with plate configuration
1 unit = radius of pole face
KF
equivalent C'
y termination
0.16
0.12
0.08
0.04
1.0
0.8
0.6
0.4
0.2
0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0

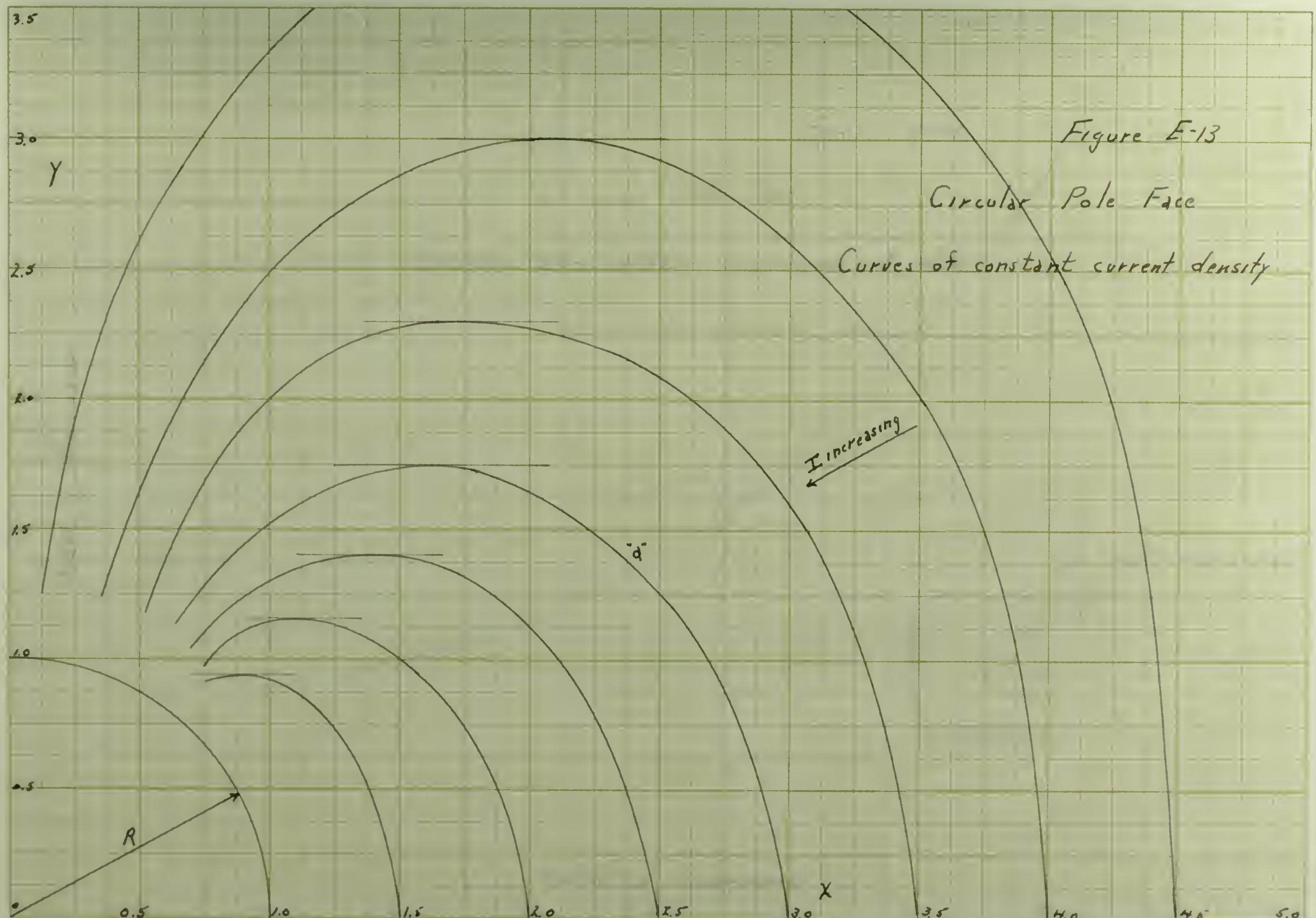
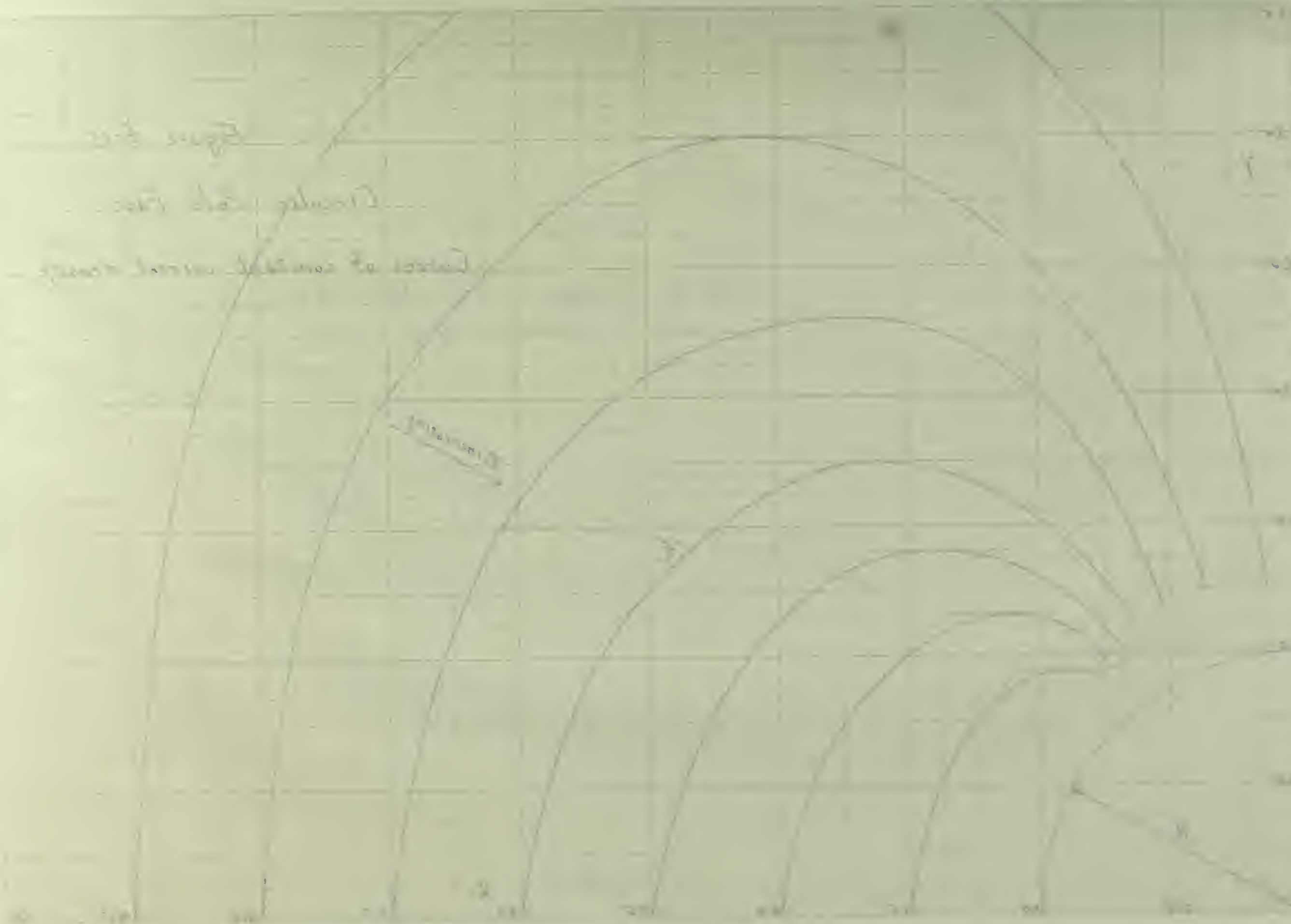


Figure E-13
Circular Pole Face
Curves of constant current density
Y
X
R
"d"
I increasing
3.5
3.0
2.5
2.0
1.5
1.0
.5
0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0


Contours of constant current density

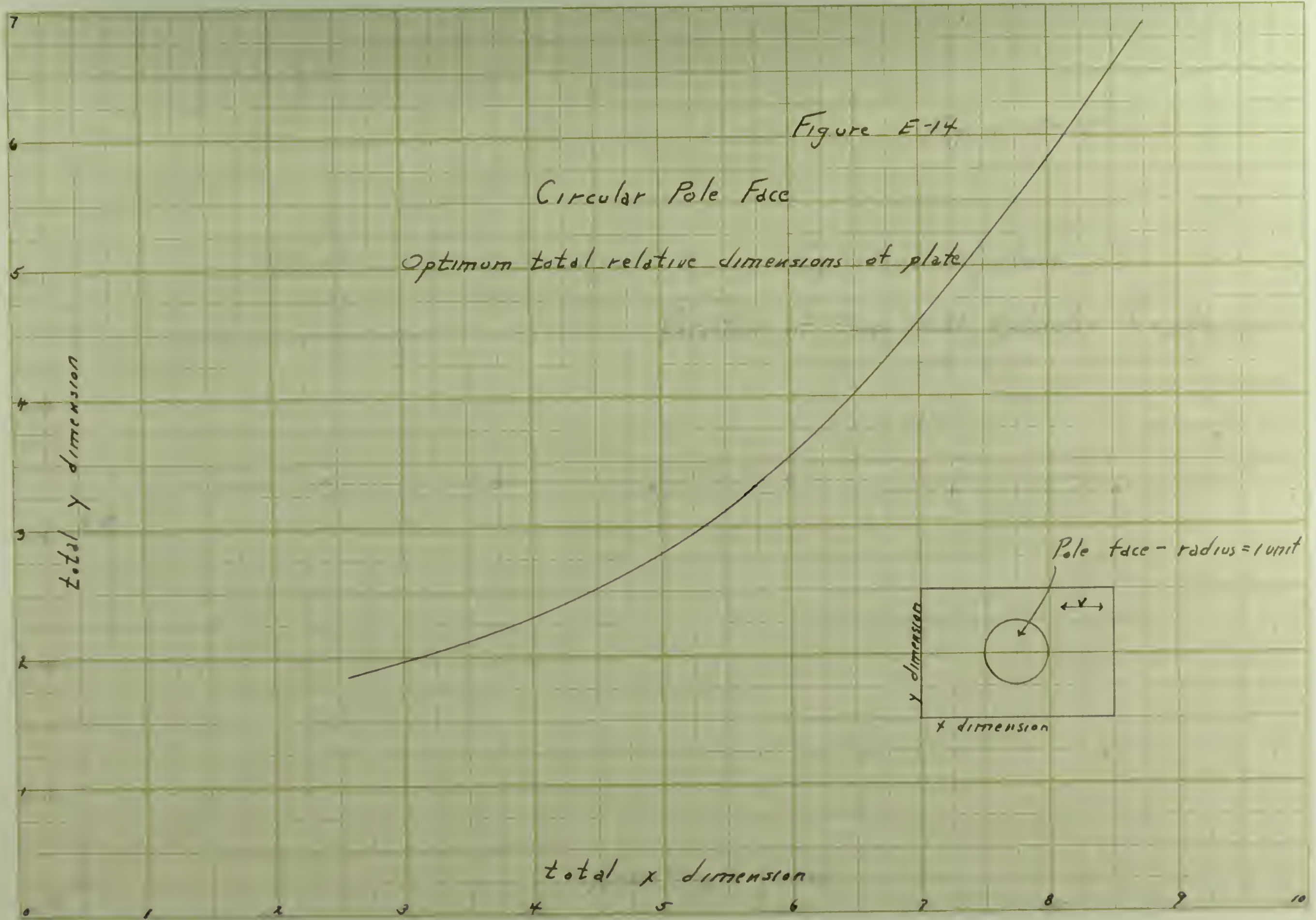


Figure E-14
Circular Pole Face
Optimum total relative dimensions of plate
Pole face - radius = 1 unit
x
x dimension
y dimension
total y dimension
total x dimension
0
1
2
3
4
5
6
7
8
9
10
1
2
3
4
5
6
7

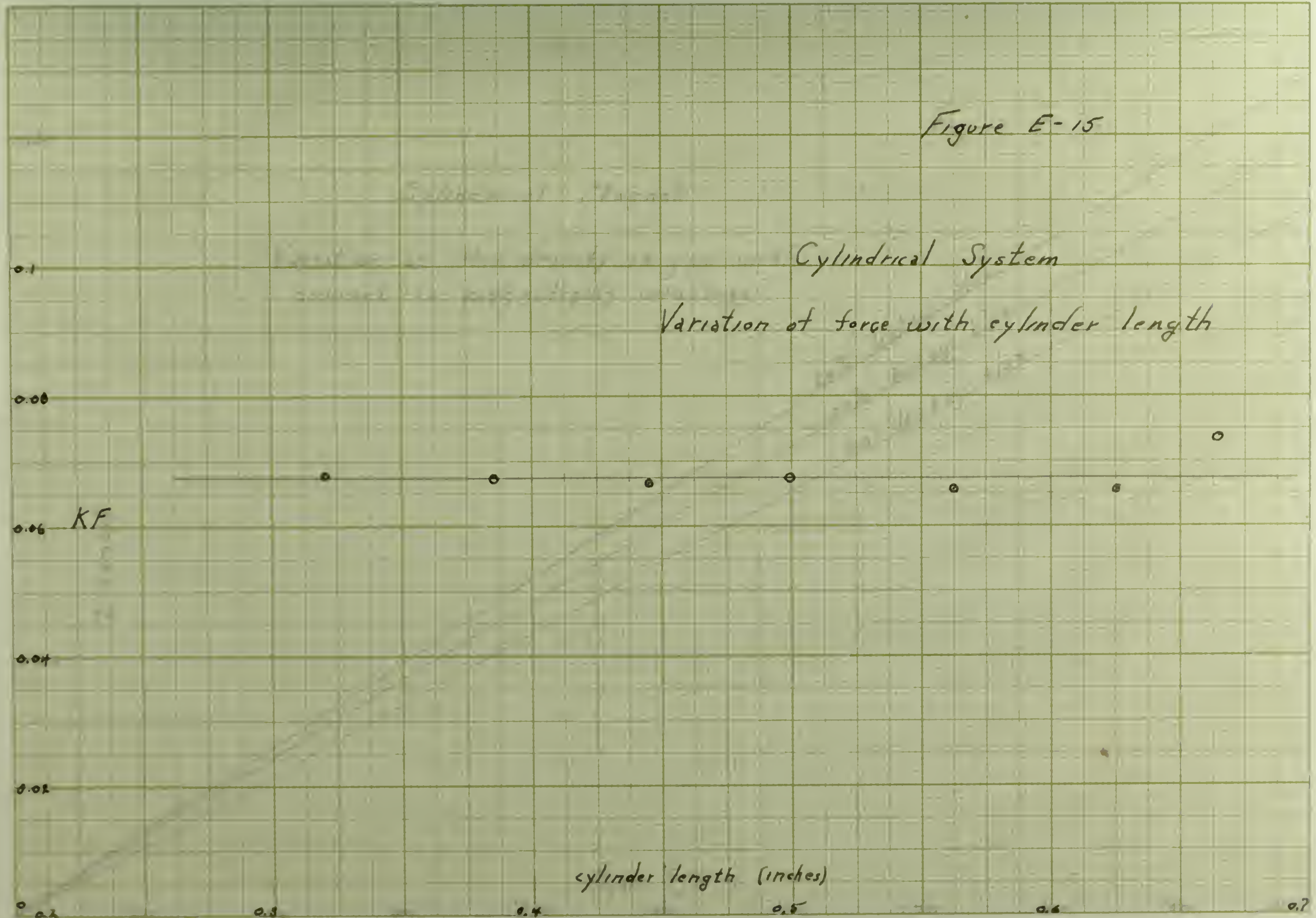


Figure E-15
Cylindrical System
Variation of force with cylinder length
KF
0.1
0.08
0.06
0.04
0.02
0
0.2
0.3
0.4
0.5
0.6
0.7
cylinder length (inches)

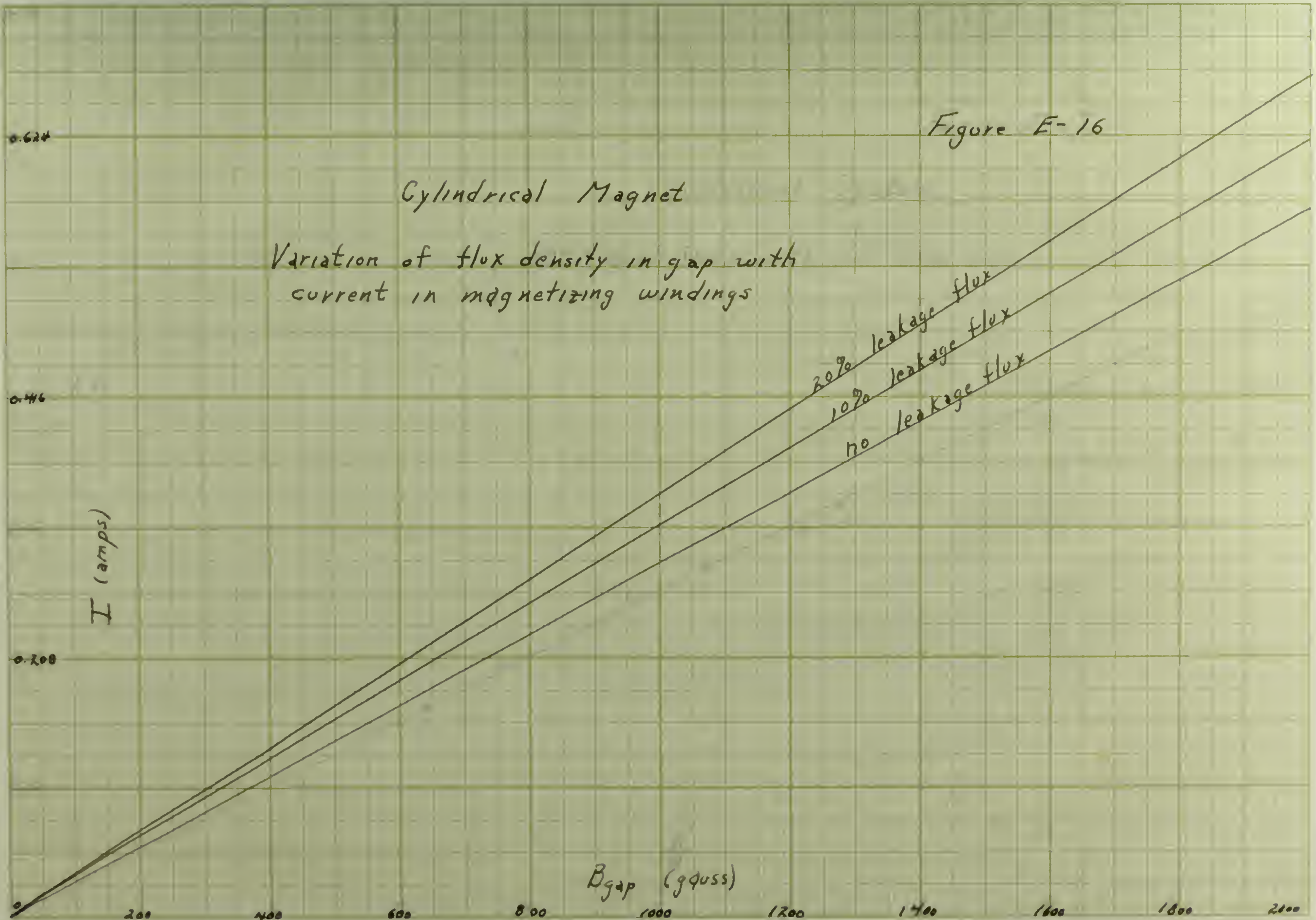
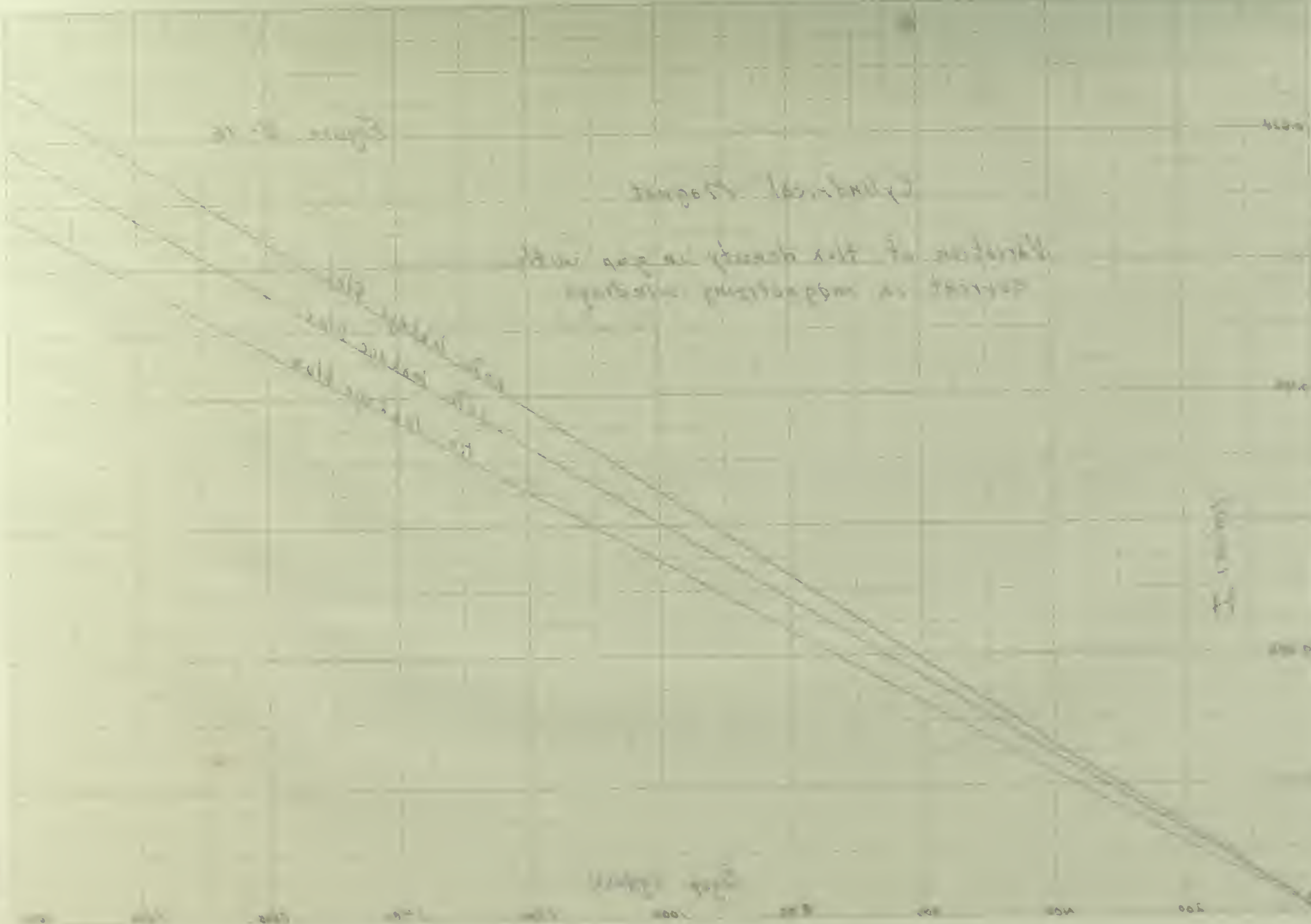


Figure E-16
Cylindrical Magnet
Variation of flux density in gap with current in magnetizing windings
20% leakage flux
10% leakage flux
no leakage flux
I (amps)
B_{gap} (gauss)
0.624
0.416
0.208
0
200
400
600
800
1000
1200
1400
1600
1800
2100


Cylindrical Magnet
Variation of flux density in gap with current in magnetizing winding

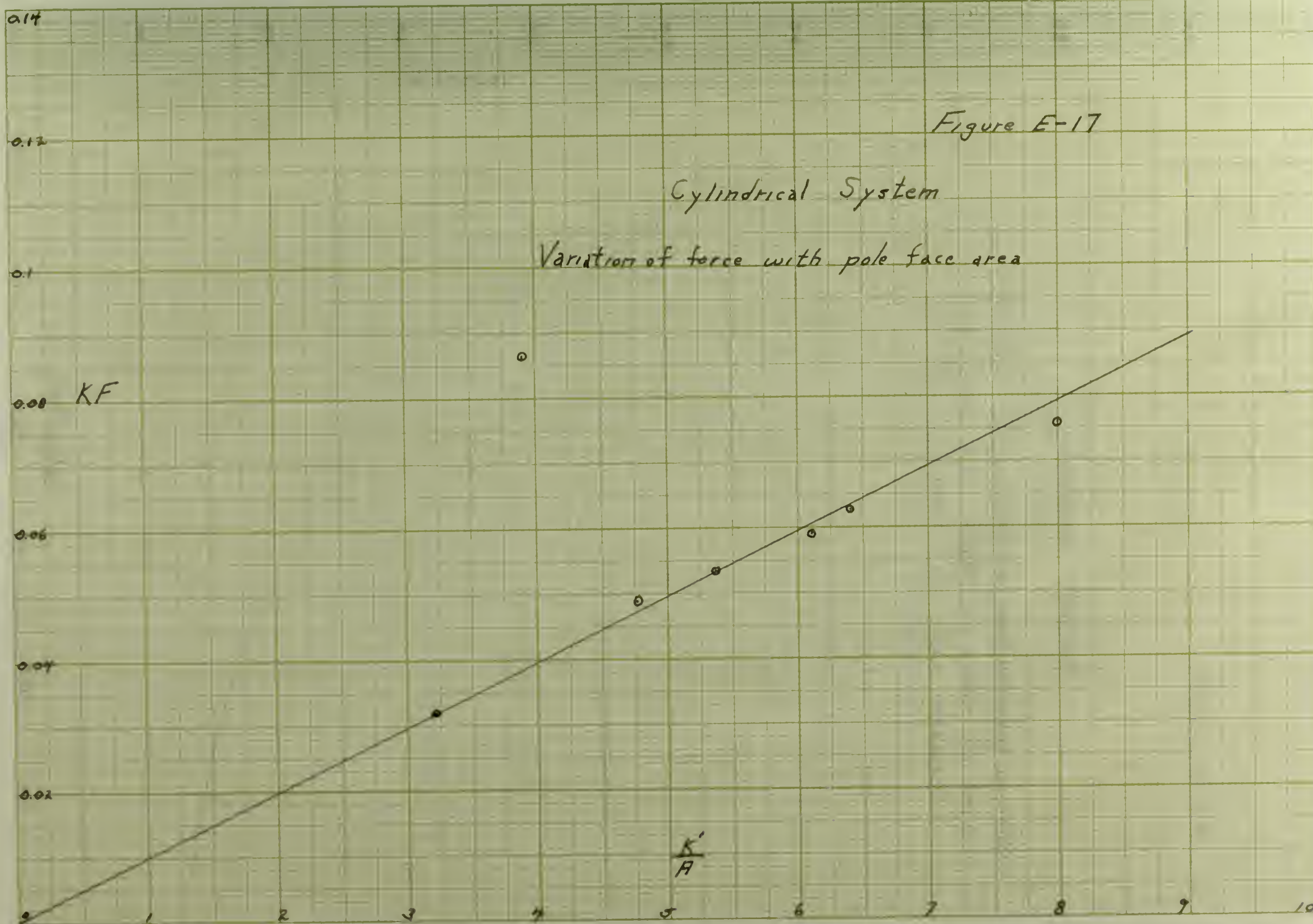
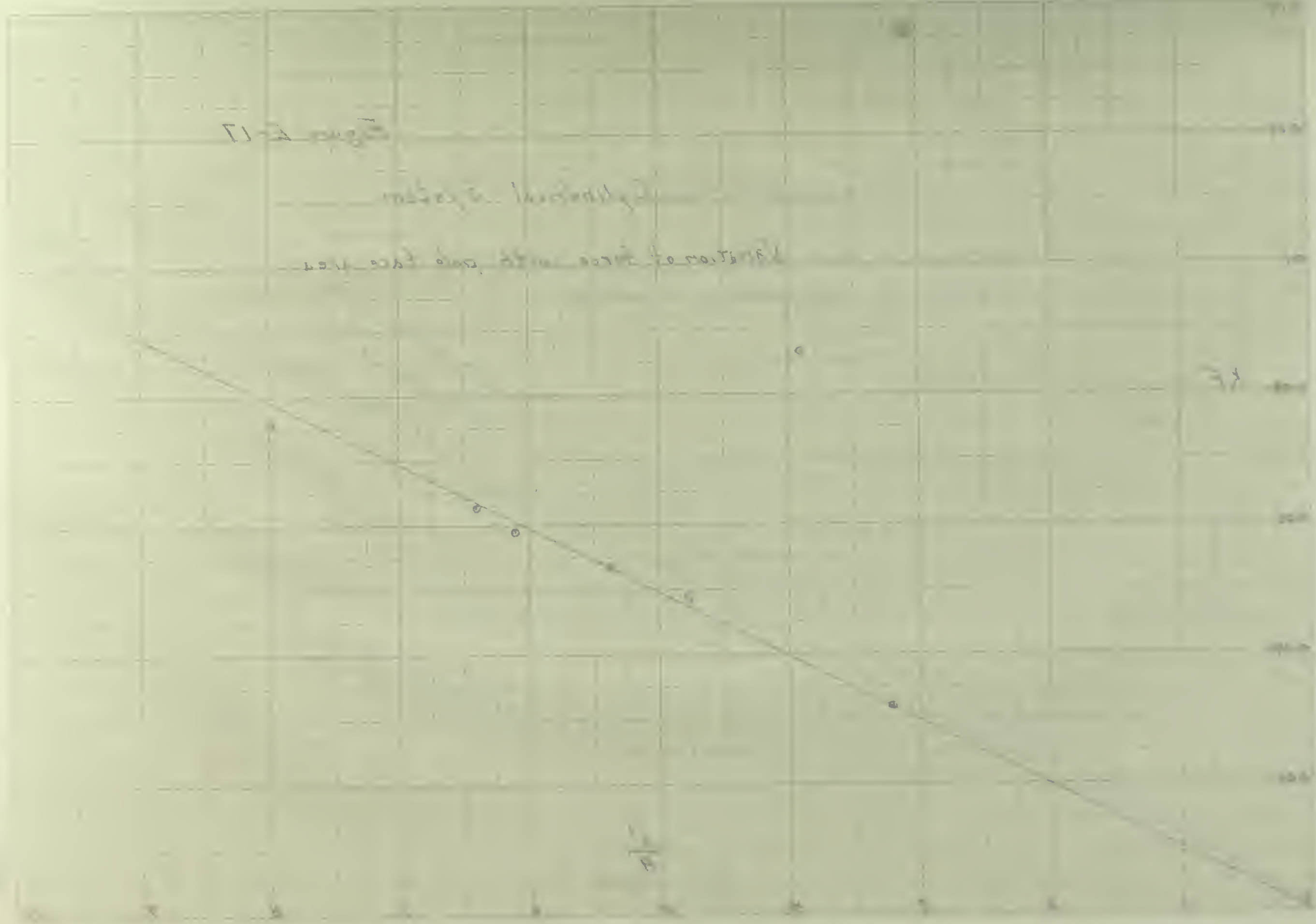


Figure E-17
Cylindrical System
Variation of force with pole face area
KF
K'/A
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
1
2
3
4
5
6
7
8
9
10



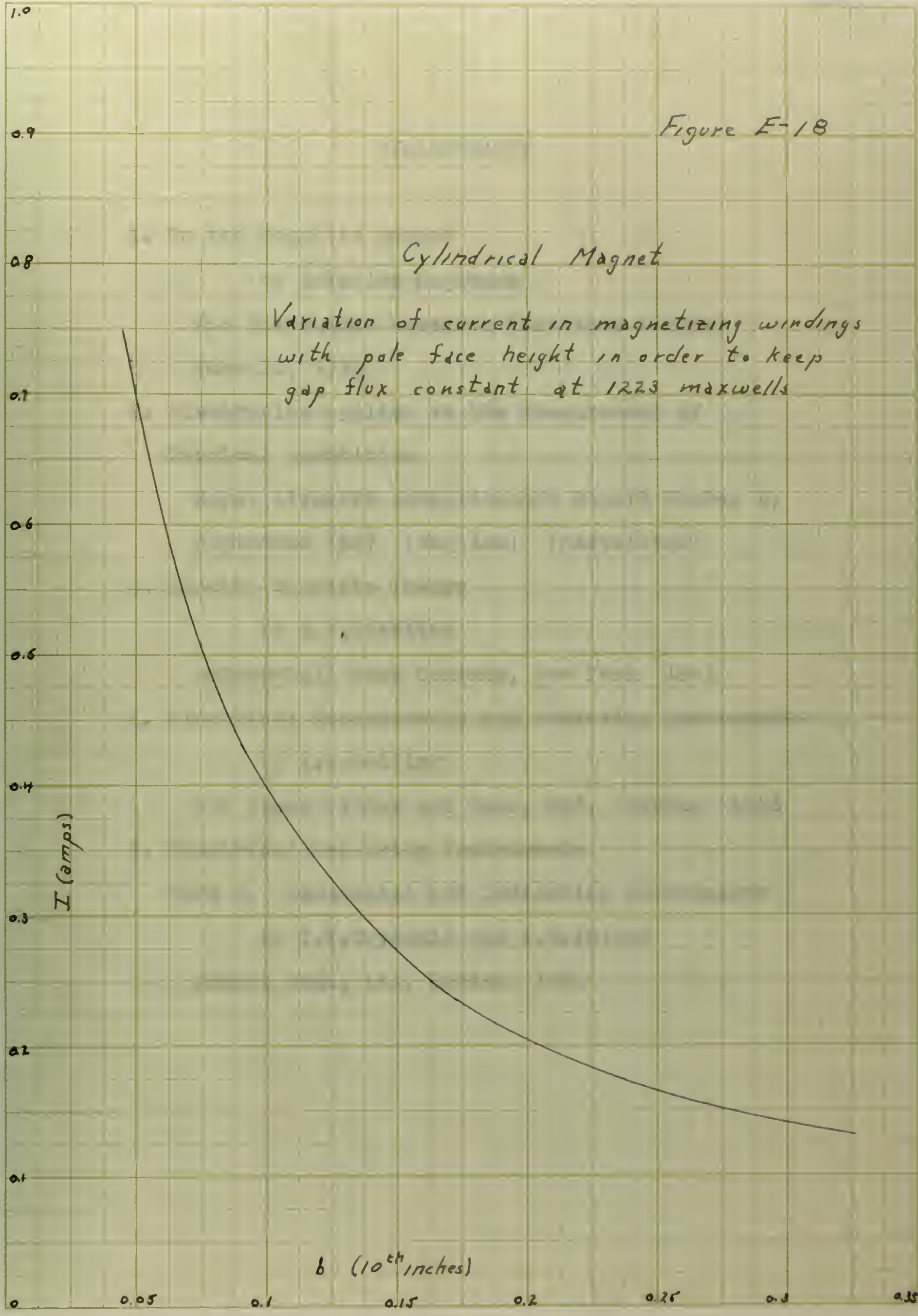


Figure E-18

Cylindrical Magnet

Variation of current in magnetizing windings with pole face height in order to keep gap flux constant at 1223 maxwells

BIBLIOGRAPHY

1. On the Magnetic Damper
by Takahiro Hagiwara
The Earthquake Research Institute (Japan)
June 20, 1934

2. Electronics Applied to the Measurement of Physical Quantities
Royal Aircraft Establishment Report Number 1, September 1947 (English) (Restricted)

3. Electro Magnetic Theory
by J.A.Stratton
McGraw-Hill Book Company, New York 1941

4. Electrical Measurements and Measuring Instruments
by A.W.Golding
Sir Isaac Pitman and Sons, Ltd. London 1946

5. Electrical Measuring Instruments
Part I. Commercial and Indicating Instruments
by C.V.Drysdale and A.C.Jolley
Ernest Benn, Ltd. London 1924

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	Attended U.S.Naval Post Graduate School at Annapolis, Maryland for one year, 1946-1947, in the School of Ordnance Engineering.
	Attended The Johns Hopkins University for two years, 1947-1949, in the School of Electrical Engineering.
Occupation:	Naval Officer, United States Navy.